Statistics II — Formula Sheet

Bachelor's Degree in Management · Academic Year 2025/2026

Probability Models

Discrete Probability Models

Binomial Distribution

$$X \sim \text{Binomial}(n,\, p)$$

$$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}, \quad x = 0, 1, \dots, n$$

$$E(X) = np \qquad V(X) = npq \qquad q = 1 - p$$

Hypergeometric Distribution

$$X \sim \text{Hypergeometric}(N,\, M,\, n) \qquad p = \frac{M}{N}$$

$$P(X = x) = \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}, \quad \max(0,\, M+n-N) \le x \le \min(M,\, n)$$

$$E(X) = np \qquad V(X) = npq\,\frac{N-n}{N-1}$$

Poisson Distribution

$$X \sim \text{Poisson}(\lambda), \quad \lambda > 0$$

$$P(X = x) = \frac{e^{-\lambda}\,\lambda^x}{x!}, \quad x = 0, 1, 2, ...$$

$$E(X) = V(X) = \lambda$$

Geometric Distribution

$$X \sim \text{Geometric}(p), \quad 0 \le p \le 1$$

$$f(x) = p(1-p)^{x-1}, \quad x = 1, 2, ... \qquad F(x) = 1 - (1-p)^k, \quad k \le x < k+1$$

$$E(X) = \frac{1}{p} \qquad V(X) = \frac{1-p}{p^2}$$

Continuous Probability Models

Uniform Distribution

$$X \sim \text{Uniform}(a,\, b), \quad a, b \in \mathbb{R},\ a < b$$

$$F(x) = \begin{cases} 0 & x < a \\ \dfrac{x-a}{b-a} & a \le x < b \\ 1 & x \ge b \end{cases}$$

$$E(X) = \frac{a+b}{2} \qquad V(X) = \frac{(b-a)^2}{12}$$

Exponential Distribution

$$X \sim \text{Exponential}(\lambda), \quad \lambda > 0$$

$$F(x) = \begin{cases} 0 & x \le 0 \\ 1 - e^{-\lambda x} & x > 0 \end{cases}$$

$$E(X) = \frac{1}{\lambda} \qquad V(X) = \frac{1}{\lambda^2}$$

Normal Distribution

$$X \sim \text{Normal}(\mu,\, \sigma), \quad \mu \in \mathbb{R},\ \sigma > 0$$

$$E(X) = \mu \qquad V(X) = \sigma^2$$

$$Z = \frac{X - \mu}{\sigma} \sim \text{Normal}(0,\, 1) \qquad E(Z) = 0 \qquad V(Z) = 1$$

$$\Phi(z) = P(Z \leq z) \qquad \Phi(-z) = 1 - \Phi(z)$$

***t*-Student Distribution**

$$T \sim t_{(n)}, \quad n \text{ degrees of freedom}$$

$$E(T) = 0 \qquad V(T) = \frac{n}{n-2}, \quad n > 2$$

Chi-Square Distribution

$$Q \sim \chi^2_{(n)}, \quad n \text{ degrees of freedom}$$

$$E(Q) = n \qquad V(Q) = 2n$$

$$Y = \sum_{i=1}^{n} Z_i^2 \sim \chi^2_{(n)}, \quad Z_i \sim \text{Normal}(0,\, 1) \text{ i.i.d.}$$

Sampling

Sample Statistics

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$$

$$S^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X})^2 = \frac{1}{n}\sum X_i^2 - \bar{X}^2$$

$$S'^{\,2} = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2 = \frac{1}{n-1}\sum X_i^2 - \frac{n}{n-1}\bar{X}^2$$

$$S^2 = \frac{n-1}{n}\,S'^{\,2} \qquad S'^{\,2} = \frac{n}{n-1}\,S^2$$

Sampling Distributions and Confidence Intervals for μ

#	Population	Distribution of $\bar{X}$	$(1-\alpha)\times 100\%$ **Confidence Interval**
1	$X \sim N(\mu,\sigma)$; σ^2 **known**	$Z = \dfrac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim N(0,1)$	$\left(\bar{x} - z_{\alpha/2}\dfrac{\sigma}{\sqrt{n}},\ \bar{x} + z_{\alpha/2}\dfrac{\sigma}{\sqrt{n}}\right)$
2	$X \sim N(\mu,\sigma)$; σ^2 **unknown**	$T = \dfrac{\bar{X}-\mu}{S'/\sqrt{n}} \sim t_{(n-1)}$	$\left(\bar{x} - t_{\alpha/2}\dfrac{s'}{\sqrt{n}},\ \bar{x} + t_{\alpha/2}\dfrac{s'}{\sqrt{n}}\right)$
2b	$X \sim N(\mu,\sigma)$; σ^2 **unknown**, $n \geq 30$	$Z = \dfrac{\bar{X}-\mu}{S'/\sqrt{n}} \dot{\sim} N(0,1)$	$\approx \left(\bar{x} - z_{\alpha/2}\dfrac{s'}{\sqrt{n}},\ \bar{x} + z_{\alpha/2}\dfrac{s'}{\sqrt{n}}\right)$
3a	$X \sim ?$; σ^2 **known**, $n \geq 30$	$Z = \dfrac{\bar{X}-\mu}{\sigma/\sqrt{n}} \dot{\sim} N(0,1)$	$\approx \left(\bar{x} - z_{\alpha/2}\dfrac{\sigma}{\sqrt{n}},\ \bar{x} + z_{\alpha/2}\dfrac{\sigma}{\sqrt{n}}\right)$
3b	$X \sim ?$; σ^2 **unknown**, $n \geq 30$	$Z = \dfrac{\bar{X}-\mu}{S'/\sqrt{n}} \dot{\sim} N(0,1)$	$\approx \left(\bar{x} - z_{\alpha/2}\dfrac{s'}{\sqrt{n}},\ \bar{x} + z_{\alpha/2}\dfrac{s'}{\sqrt{n}}\right)$

Sampling Distribution and Confidence Interval for p

Population	Distribution of $\hat{p} = X/n$	$(1-\alpha)\times 100\%$ **Confidence Interval**
$X \sim \text{Binomial}(n,p)$, $n \geq 30$	$\dfrac{\hat{p}-p}{\sqrt{\hat{p}(1-\hat{p})/n}} \dot{\sim} N(0,1)$	$\left(\hat{p} - z_{\alpha/2}\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z_{\alpha/2}\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\right)$ *(approx.)*

Sampling Distribution and Confidence Interval for σ^2

Population	Distribution of S'^2	$(1-\alpha)\times 100\%$ Confidence Interval
$X \sim N(\mu, \sigma)$	$Q = \frac{(n-1)S'^2}{\sigma^2} \sim \chi^2_{(n-1)}$	$\left(\frac{(n-1)s'^2}{q_{\text{sup}}},\ \frac{(n-1)s'^2}{q_{\text{inf}}}\right)$

$$P(Q > q_{\text{inf}}) = 1 - \frac{\alpha}{2} \qquad P(Q > q_{\text{sup}}) = \frac{\alpha}{2}$$

Parametric Hypothesis Tests (at level α)

Hypotheses	Test Statistic (TS)	Critical Region
$H_0 : \theta = \theta_0$ vs $H_1 : \theta \neq \theta_0$	TS_0	$(-\infty,\, q_{\alpha/2}) \cup (q_{1-\alpha/2},\, +\infty)$
$H_0 : \theta \leq \theta_0$ vs $H_1 : \theta > \theta_0$	TS_0	$(q_{1-\alpha},\, +\infty)$
$H_0 : \theta \geq \theta_0$ vs $H_1 : \theta < \theta_0$	TS_0	$(-\infty,\, q_{\alpha})$

$P(\text{TS} > q_\alpha) = \alpha$ Reject H_0 if $\alpha > p$-value

$\alpha = P(\text{Reject } H_0 \mid H_0 \text{ true})$ (Type I error probability)