

Wealth and the Principal-Agent Matching*

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Abstract

I study the role the agent's wealth plays in the principal-agent matching with moral hazard and limited liability. I consider wealth and talent as the agent's type, and size as the firm's (principal's) type. Wealthier agents match with bigger firms, when talent is homogeneous among them, whereas for equally wealthy agents, more talented agents will match with bigger firms. I describe economic conditions over types such that pairs of higher types will write contracts in which the agent gets more than the limited liability rents. Finally, I write conditions on wealth for assortative matching in talent to hold.

JEL-Classification: D86, D82, C78, J33, M12.

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1 Introduction

Wealth can play a major role in the design of optimal incentives, and therefore in the matching between principals and agents. In order to study the compensation of these agents, it is fundamental to understand how they will endogenously match with the firms they end up working for,¹ and the specific role that the characteristics of the contracting parties play in it. I propose a model in which risk neutral agents, characterized by their wealth and talent, match with firms (principals), characterized by their size, to perform a task and in the presence of moral hazard. This model allows to study the contracts being signed by the parties, and the implications that wealth brings to the matching for the traditional results of positive assortative matching between talented agents and bigger firms.

Wealth can affect the agent's behavior, and the optimal contract, in two different ways: through the agent's risk aversion or through limited liability. While the scarce literature studying the effects in compensation focused on the former (Chade and de Serio, 2014, Thiele and Wambach, 1999), this article concentrates on the latter. Limited liability prevents the principal from selling a high participation in the company to the agent in order to achieve an output closer to the one without information asymmetries.² Having this in mind, I raise the question: How do principals and agents match? In particular: Do wealthier agents match with larger firms or the opposite? Can wealth be assigned to agents in such a way that the positive assortative matching of talented agents working in bigger companies does not hold?

The empirical literature on compensation, has started to incorporate some degrees of matching and moral hazard. Of particular relevance are the works of Gabaix and Landier (2008) and Terviö (2008), which provide interesting conclusions relating the variables that might drive the matching between executives and firms. Gabaix and Landier (2008) present a model of CEO and firms matching, based on the distribution of the CEOs' talent. They assume that compensation is based on talent, while firms and CEOs are differentiated between them in size and talent respectively. Their main theoretical contribution is how compensation on talent reacts to the talent distribution, reaching the conclusion (supported by their data) that even highly similar talented CEOs can show big differences in their wages. They also conclude that bigger firms lead to higher wages in equilibrium. Terviö (2008) on the other hand, develops a matching model that tries to obtain the distribution of CEO's ability from the known distribution of pay and firm's market value.

¹Akerberg and Botticini (2002) shows the pervasive effects of neglecting the endogenous matching when studying incentive contracts empirically.

²Shetty (1988) studied the effect that limited liability has on the contracts between tenants and landlords.

The author concludes, among other important insights, that the wide differences in compensation for a small distribution of CEO's ability is given by firms characteristics. It happens as well that competitive factors are crucial to explain the huge differences in compensation levels among managers. Again, Terviö (2008) neglects moral hazard problems, and neglect any impact wealth might have on the the design of incentives. An important lesson though is the effects competition has in the level of wages. The model introduced in this article provides a theoretical ground for these findings. The more similar the agents and firms, the latter needs to pay more to keep the high type agents agents, otherwise, smaller firms could outbid the bigger ones for them. This competitive effect generates efficiency gains, which reflects in higher expected surplus.

Edmans, Gabaix and Landier (2009) expands the framework of Gabaix and Landier (2008), by incorporating the agency problems within a competitive assignment model. They conclude that the compensation, as a share of the firm, is decreasing in firm size, and they evaluate the effectiveness of incentive compensation, a result that can also be replicated in the model introduced in this article. Moreover, in their conclusions they raise a question that relates directly with this article: Are CEO incentives increasing in wealth? They refer to this problem and the impossibility to solve, at least empirically, given that there is no information on absolute wealth for the CEO, and it is only possible to obtain the wealth inside the firm, in terms of stocks and options. This is one of the things that Baker and Hall (2004) tried to address. They were trying to investigate the relationship between CEO's compensation with the firm's size. To do this they developed a single and multitask agency problem, and find the optimal compensation scheme for the CEO. They later estimate their model focusing on understanding how the marginal effect of a manager depends on the firm's size. An interesting finding is that the CEO's bonus decreases, as a share of the company, with the firm's size. They do not use a matching model as the previous article, nevertheless this result is confirmed by the model presented in here. The authors use wealth to determine the risk aversion only, given a firm size, and later make three assumptions that allow them to proxy wealth in three different ways: first they assume that wealth is proportional to total annual compensation, later they assume that wealth is the CEO's holding in the firm (options plus stock), and finally they assume that CEOs of big firms aren't richer nor poorer than CEOs of smaller firms. For us, this is not the case, as the agent's wealth is key to determine 1) if the agent is cash constrained, and 2) if it is cash constrained, how much limited liability rents can the agent extract.³

³Limited liability rent stands for the rents the agent can extract thanks to the fact his action is hidden from the principal while having limited liability.

Macho-Stadler and Pérez-Castrillo (2021) utilizes the competitive matching between principals and agents framework to study how the final equilibrium is affected by the different dimensions in which the participants in the market can differ: principals can have more or less risky projects, while agents have different risk aversion, skills, and also can affect the risk of the project differently. They later expand their analysis to the situation in which principals can hire more than one agent, and with repeated interactions between principals and markets to study the optimal contract length. A substantial difference with this paper is that they consider that utility is perfectly transferable between principals and agents, a situation that, as I present later in this article, is not necessarily the case when moral hazard is introduced.

The core of the empirical literature studying the matching between principals and agents has focused on the agent's talent as the main driver, and when it has focused on wealth it has been as an opportunity cost that reflects on the cost of effort (Edmans et al., 2009) or as a proxy for risk aversion (e.g. Baker and Hall, 2004, Bandiera, Guiso, Prat and Sadun, 2015). Wealth, however, through the limited liability can play a major role.⁴ In this article, I try to study precisely that effect, and to capture that, in the model used here, I shut down any other channel in which wealth could play a role.

It is a standard result in the moral hazard literature that under asymmetric information the principal can achieve the first best level of effort with a risk neutral agent that is not cash constrained, allowing the principal to define negative transfers to the agent in case of bad outcomes, or as the traditional interpretation indicates, an up-front cash payment from the agent to the principal. This can be observed for example in the franchise model, or in the cab business. This scenario is not common in the corporate world (see Baker, Jensen and Murphy, 1988). This can be due to the lack of wealth from the agents to make the transfers, legal or social limitations, or it can be simply interpreted in a different manner, for example as an obligation for the executives to buy shares from the company. Other industries, however, do face these kind of contracts more explicitly. For example, lawyers, when promoted to partners, must buy their partnership with money from their own pockets. Physicians, in some countries, must buy a participation in the clinics where they receive their patients. In general any kind of partnership will involve

⁴Dam and Perez-Castrillo (2006), to my knowledge, is the only that explicitly considered the role of wealth in a traditional principal-agent matching setup. However, they consider homogeneous principals, while I allow for heterogeneous principals. I also allow for agents to differ in talent. Legros and Newman (1996) also consider a problem of matching agents of the same type with different wealth levels, that get together to form firms between them. Serfes (2005) study a matching problem with moral hazard, with heterogeneity being risk aversion for agents as principals. Finally, more recently Lindenlaub (2017) studies a bi-dimensional matching problem with different types of skills instead of the agent's wealth.

transfers from the agents to the principal.

As mentioned, in the absence of limited liability the principal can set up a contract that achieves a first best outcome, by selling the firm to the agent for the whole expected surplus leaving the agent without any limited liability rents. This is no longer true once the agent is cash constrained. I focus my study on how this channel affects the matching between agents and principals. In a first stage I study the isolated version of the principal-agent model, finding that the principal's utility increases in matches with wealthier agents, whereas the agent's utility increases by working in bigger firms. This suggests that a positive assortative matching is to be expected, that is wealthier agents should work in bigger firms.

Because utility is not perfectly transferable in the principal-agent model, as the sharing of the surplus affects the strength of the incentives, it is necessary to use the concept of generalized increasing differences, introduced by Legros and Newman (2007), in order to analyze the matching. I find that there is positive assortative matching (PAM) between principals and agents where the type for principals is the firm size, and the agents' is their wealth. The same PAM is valid when the agents' type is their talent: more talented agents work for bigger firms.

Then I describe the contracts that each pair will sign and the expected output they will produce. I also consider gains in efficiency (by generating contracts closer to the first best for some matches) that are generated by the market pressure introduced by competition, that is, the fact that other principals could offer more convenient contracts to my own agent. In particular, I provide conditions under which high-type principals give larger incentives to their agents than what they give in the isolated version of the model. For this market pressure to be effective, though, firms should have similar sizes.

Finally I give an example in which a wealthy agent has lower skills than a poor one, a situation in which generalized increasing differences may not hold anymore, and therefore, neither needs to hold the positive assortative matching between principals and agents when considering talent as the agent's type. I also provide a sufficient condition on the distribution of talent for the positive assortative matching between firm (in size) and agent (talent) to hold independently of the distribution of wealth. The situation in which there is no clear positive correlation between wealth and skills is not unexpected according to the literature, for example Becker (2006) concludes, among other things, that wealth is unlikely to proxy for skill, and Terviö (2008) finds that even the wide observed disparities in compensation for top companies executives could be explained assuming very narrow distributions of talent, and therefore it might be the case that talent is not necessarily correlated with wealth. This situation can also apply to agents in the beginning of their

careers, where they have not had time to reveal their skills and accumulate wealth. As a consequence, the assumptions made on the correlation between wealth and talent are critical to claim that, for example, there is positive assortative matching between bigger companies and more talented agents.

This article differs from the existing literature by focusing on the role limited liability has on the competitive matching between principal and agents, while allowing heterogeneous principals (on the size of their project) and, at the same time, heterogeneous agents (skills and wealth). The results obtained, in particular that wealth could compromise the standard assumption of positive assortative matching between firm size and agent's talent, stresses the importance of gathering data on wealth, or in its absence on the assumptions made about it, when studying models that link firms and agents, and the potential implications on the outcomes, in particular after the findings of Akerberg and Botticini (2002) on the importance of accounting for the endogenous matching between principals and agents on empirical studies.

In Section 2 I describe the baseline principal-agent model I use in this article, to proceed with the analysis of the utility possibility frontier and the matching analysis. In Section 3 I provide an example of a distribution of wealth and talent in which the positive assortative matching does not hold. Finally, I conclude in Section 4.

2 The Model

In this section I develop a slightly modified version of the classical model of moral hazard with risk neutral agents.⁵ The modifications I introduce allow to capture the effects of different levels of cash constraint for the agent and the size of the firm in the optimal compensation scheme and welfare allocation, by parameterizing the model in the firm's size, agent's wealth, and the agent's talent.

Both principal and agent are assumed to be risk neutral. I assume that the firm's output x can have two states, $x \in \{0, \xi\}$, where $\xi \in [0, 1]$ reflects the firm's size.⁶ The agent's effort e can be chosen between 0 and 1, and affects positively the probability of success — having $x = \xi$ as the outcome — in the following way: $Pr(x = \xi|e) = p(e) = e$.⁷ Let a and b be the base wage

⁵See Bolton and Dewatripont (2005), Laffont and Martimort (2002), and Salanié (2005) for examples.

⁶Considering ξ as a parameter allows to use it as firm size. For example in Gabaix and Landier (2008), the authors mention that variables such as earnings or capitalization can be considered as firm size, and Bandiera et al. (2015) consider the number of employees as firm size. Given the characteristics of this single-period model, defining size as the earnings in the good state avoids the inclusion of more variables that would needlessly complicate the model.

⁷The parameterization of the model in the other variables makes the constraint for effort, to be

and the bonus respectively. Although a is paid independently of the outcome, b is paid by the principal only if $x = \xi$ is observed. Effort is costly for the agent, and is represented by the cost function $c(e) = e^2/(2\tau)$. Here $\tau \in (0, 1)$ measures the ability - or talent - of the agent: a higher τ implies lower effort cost. Assume that the reservation utility for principal and agent are 0 and \bar{u} respectively.⁸ Let \bar{u} be the maximum between 0 and whatever he can get by working somewhere else. Also assume that the agent has personal wealth $\omega \geq 0$. The agent's cash constraint is therefore determined by $-\omega$, meaning that the principal can never set wages such that the agent transfers more than ω to her.

First, the principal makes a take-it-or-leave-it offer to the agent, specifying the base wage and the bonus for success; the agent accepts or rejects the offer; conditional on accepting it, he decides how much effort to exert. Finally the outcome is realized and the principal makes the transfer to the agent.

The maximization problem of the principal is given by:

$$\begin{aligned} \max_{e,a,b} \quad & -a + p(e)[\xi - b] & (1) \\ \text{s.t.} \quad & a + p(e)b - c(e) \geq \bar{u} & (\text{PC}) \\ & e \in \arg \max_{\hat{e}} \{a + p(\hat{e})b - c(\hat{e})\} & (\text{IC}) \\ & a \geq -\omega & (\text{CC}) \end{aligned}$$

Equation (PC) is the participation constraint that ensures that the agent will accept the contract proposed by the principal.⁹ Equation (IC) represents the incentive compatibility constraint that ensures that the agent will choose endogenously what the principal has chosen as optimal effort, and finally (CC) represents the cash constraint. Usually in the literature (CC) is represented as $a \geq 0$ to model a cash-constrained agent, or in other words, a situation with limited liability. In this article, instead, we allow for different levels of wealth.

The solution to this problem depends on the value that ω takes. In fact, it is well known in the moral hazard literature that if ω is high enough (and this will be shown in the model as well), the principal can achieve first best effort with a contract that is equivalent to selling up-front the outcome to the

lower than 1, not binding.

⁸By the nature of the problem, I consider the reservation utility for the agent to be the minimum change in utility he is willing to accept. Note that if the reservation utility \bar{u} is included in the participation constraint, for the risk neutrality, it can be canceled out.

⁹The participation constraint would be indeed $\omega + a + p(e)b - c(e) \geq \bar{u} + \omega$, as the agent's utility depends on his own wealth and on his outside option, but ω cancels out on both sides.

agent.¹⁰ Doing so, he can extract the whole expected surplus, minus \bar{u} . On the other hand, if ω is too low (lower than $\frac{\xi^2\tau}{4}$) then the principal is better off just giving away the project. An interpretation of this negative ω could be the existence of a minimum wage.

When solving the problem not all the constraints are going to be binding. If the (CC) is too severe, then the (PC) is not binding. Conversely if the agent has large wealth, and/or a high reservation utility, one finds that only the (PC) is binding. There are intermediate cases in which both constraints are binding, and the solution is given by the system of equations provided by the constraints of the problem. Because this model is quite standard in the literature I leave its detailed derivation for Appendix B.2. Solving this model, though, provides us with the utility possibility frontier (Table 1),¹¹ which is extremely useful to solve the matching. I leave its detailed derivation of the model can be found in Appendix B.2. The utility of the principal is represented by v , whereas the agent's utility is represented by u .

Variable	Binding Constraint		
	(CC) $\omega + \bar{u} < \frac{\xi^2\tau}{8}$	(CC) and (PC) $\frac{\xi^2\tau}{8} \leq \omega + \bar{u} \leq \frac{\xi^2\tau}{2}$	(PC) $\frac{\xi^2\tau}{2} < \omega + \bar{u}$
e	$\frac{\xi\tau}{2}$	$\sqrt{(\bar{u} + \omega)2\tau}$	$\xi\tau$
a	$-\omega$	$-\omega$	$\bar{u} - \frac{\xi^2\tau}{2}$
b	$\frac{\xi}{2}$	$\frac{\sqrt{(\bar{u} + \omega)2\tau}}{\tau}$	ξ
$E[u]$	$\frac{\xi^2\tau}{8}$	$\bar{u} + \omega$	$\bar{u} + \omega$
$E[\Delta u]$	$-\omega + \frac{\xi^2\tau}{8}$	\bar{u}	\bar{u}
$E[v]$	$\omega + \frac{\xi^2\tau}{4}$	$\xi\sqrt{(\bar{u} + \omega)2\tau} - 2\bar{u} - \omega$	$\frac{\xi^2\tau}{2} - \bar{u}$
Δ Surplus	$\frac{3\xi^2\tau}{8}$	$\xi\sqrt{(\bar{u} + \omega)2\tau} - \bar{u} - \omega$	$\frac{\xi^2\tau}{2}$

Table 1: Solution to problem in (1). Δu and Δ Surplus represent the induced changes in the agent's utility and total surplus respectively.

Supermodularity on surplus is often enough to have positive assortative matching (PAM) when utility is transferable. However, in the principal-agent setup, utility is not perfectly transferable. In a contract, the principal can find many ways to transfer utility to the agent, and the case of perfectly transferable utility would be simply a money transfer to the agent (a higher fixed wage a). This is not an optimal alternative for the principal. She will instead set a higher bonus b , incentivizing the agent to exert a higher level of effort, and thus increasing the surplus. The transfer of utility to the agent is

¹⁰First best refers to the situation in which there is no asymmetry of information between the agent and the principal.

¹¹Surplus stands for the aggregation of the principal and agent utilities.

financed in part by giving up a share of the surplus, but also by the increase of it, so the utility loss for the principal is lower than the gains in utility for the agent.

Legros and Newman (2007) [LN henceforward] introduced a methodology to address the matching problem when utility is not perfectly transferable. They consider a match between individuals type R and S . Let $s > s'$ and $r > r'$ be two different types within each category, and the outcome be a match between an individual from R with another individual from S . They argue that r is going to match with s if r can outbid r' for s . In particular, they introduce the concept of generalized increasing differences (GID) which relies on the utility possibility frontier (UPF) in order to determine the matching between individuals.¹²

The UPF describes the combinations of utilities u and v that are Pareto efficient.¹³ For the sake of consistency I will use the same notation for the UPF, outlined in the following definition:

Definition 1. *Let $\theta = (\omega, \tau)$ be the agent's type, and ξ be the principal's type. The utility possibility frontier is described by the following functions:*

- *Let $\phi(\xi, \theta, u)$ be the maximum utility that a principal type ξ can obtain when matching with an agent of type θ , and this agent gets utility u .*
- *Let $\psi(\theta, \xi, v)$ be the maximum utility that an agent type θ can obtain when matching with a principal of type ξ , and this principal gets utility v .*

Thus, ϕ and ψ represent the UPF from the points of view of the principal and agent respectively, and for a bijective UPF and given types, one is the inverse of the other, with respect to utility levels.

In the model with only one principal and one agent - introduced in the beginning of this section - it has been highlighted that, for the optimal contract, there are 3 possible situations: only (CC) is binding, only (PC) is binding, or both (CC) and (PC) are binding. The (IC) is always binding. In Table 1, we can also obtain the UPF, with $E[\Delta u]$ and $E[v]$.

The situation when only the (CC) is binding represents a single point on the UPF. This happens because the agent is getting a fixed rent, and the (PC) is not binding. If we want to move to the right along the UPF we need to give higher utility to the agent, and then the (PC) starts binding. The (CC) is binding until the unconditional transfer a implied by when only (PC) is binding exceeds $-\omega$ (in other words, (CC) stops binding), leading to:

¹²In Appendix A I provide a brief explanation of the mechanics of the matching process.

¹³That is, neither principal nor agent can have higher utility without the other being worse off.

$$\underbrace{\bar{u} - \frac{\xi^2\tau}{2}}_{a_{pc}} \geq \underbrace{-\omega}_{a_{cc}} \quad (2)$$

or when looking at the UPF, when $u \geq \xi^2\tau/2 - \omega$. For all the values of $u \in [\xi^2\tau/8 - \omega, \xi^2\tau/2 - \omega]$, both constraints are binding. Of course the case can appear in which $\xi^2\tau/8 \leq \omega$, or even $\xi^2\tau/2 \leq \omega$, nevertheless, as mentioned earlier, I assume that the agent cannot obtain negative expected utility, and therefore $\bar{u} \geq 0$. In particular for the UPF I assume that $\bar{u} = 0$. Note that in this case, in which the agent is risk neutral, the only relevant for him is the expected variation in utility, and not the utility level *per se*.

What is different to the usual UPFs is that this UPF's domain varies with ω , τ and ξ . For example, when ω is small, it considers values of u that are strictly positive. This represents a small variation with respect to LN's UPF, as they assume that its domain is between 0 and some upper bound for both individuals. On the other hand, a bigger company, or a more talented agent increases the expected value of the company, making an agent cash constrained, and therefore creating limited liability rents. To address this fact, I define the following correspondences:

Definition 2. *Given the UPF, define:*

- $\underline{u}_\theta(\xi) := \max\{0, \frac{\xi^2\tau}{8} - \omega\}$ as the minimum level of utility that an agent of type θ can get from a match with a principal of type ξ .
- $V_\xi(\theta) := [0, \phi(\xi, \theta, \underline{u}_\theta(\xi))]$ as the feasible utility levels that an optimal contract can give to a principal whose firm has size ξ contracting with an agent of type θ . By definition, $V_\xi(\theta)$ is also the domain of $\psi(\theta, \xi, \cdot)$.
- $U_\xi(\theta) := [\underline{u}_\theta(\xi), \frac{\xi^2\tau}{2}]$ as the set of feasible utility levels that an optimal contract can give to an agent of type θ working for a firm of size ξ . By definition, $U_\xi(\theta)$ is also the domain of $\phi(\xi, \theta, \cdot)$.

In Figure 1 I draw the UPF for different levels of ω .¹⁴ The upper dotted line represents the surplus in the first best, whereas the lower dotted line represents the surplus when only the (CC) binds. Both lines have a slope of -1 . In Figure 2 I repeat the same exercise but changing the value of τ instead.¹⁵

¹⁴Note that for the case Figure 1a, letting $\omega < 0$ is equivalent to assuming a minimum wage of $-\omega$. What happens is that the smaller ω the left dot decreases through the interior dotted line. At its lowest point the agent is keeping all the surplus, and a discontinuity of the UPF arises. Recall that being over the interior dotted line would mean the agent is still not exerting first best effort as the transfer at that point, that maximizes the principal's utility, would be on the fixed part of the wage, so when $\omega = \frac{\xi^2\tau}{4}$, the UPF would be the point $(\frac{\xi^2\tau}{2}, 0)$.

¹⁵In Figure 4, in Appendix B, I show as well how the UPF changes by changing the firms' size only.

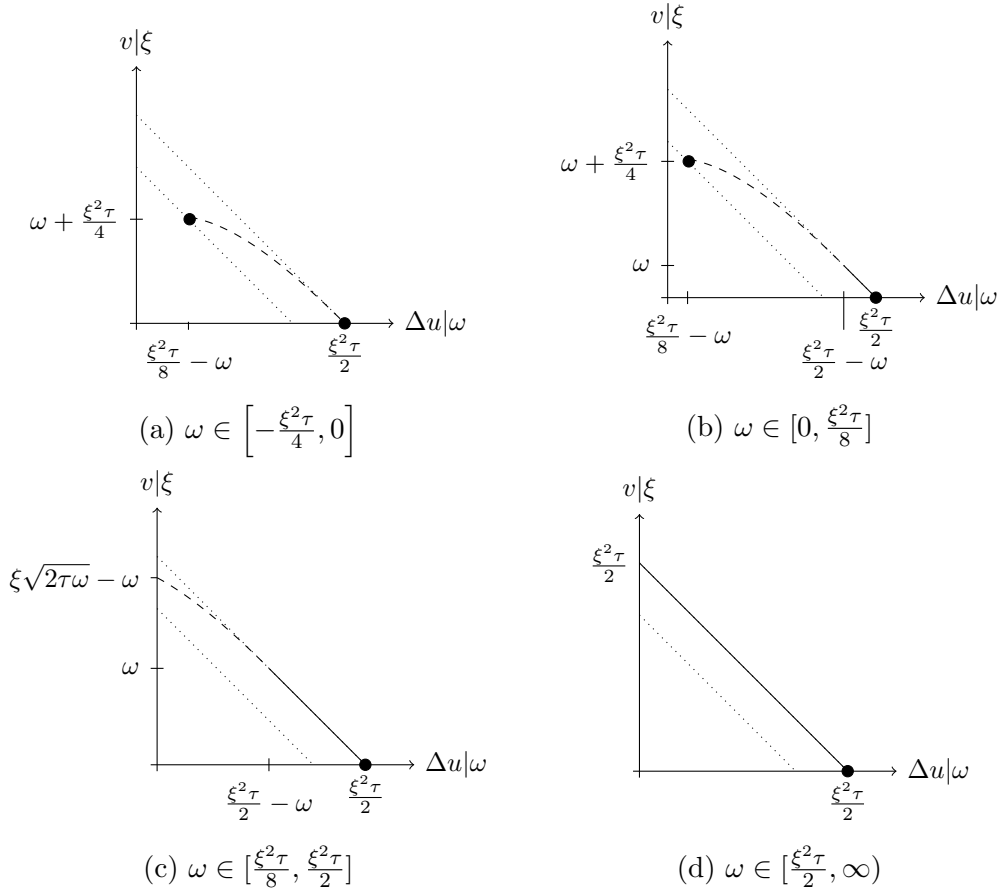


Figure 1: UPF for different levels of ω .

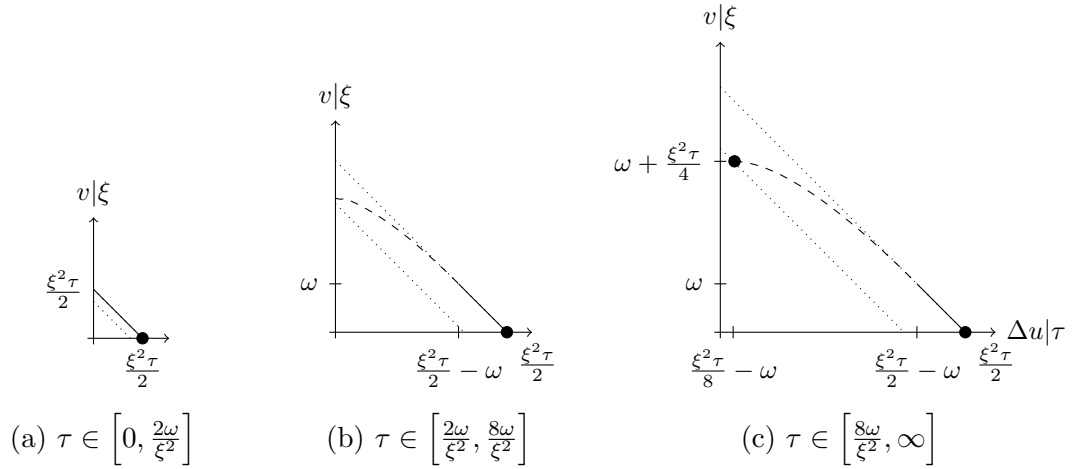


Figure 2: UPF for different levels of τ .

The left and right solid points rest over the second and first best surplus respectively. Specifically the left dot represents the contract in a second best when only the CC is binding, whereas the right dot represents the principal giving away the firm for free to the agent. The solid line represents sections of the UPF when the principal has sold the firm to the agent in exchange of some fixed fee. The dashed line represents the UPF when both constraints, CC and PC, are binding.

By increasing the agent's wealth, the UPF expands upward until it reaches the first best surplus. As the utility for the agent increases, the principal implements the contract indicated by the system of the three constraints CC, PC, and IC. At the same time, the higher the utility for the agent, the more relevant the PC becomes compared to the CC, and therefore the solution gets closer to the first best outcome. Here, the principal is selling cheaper a share in the outcome and therefore the agent, for the same price, is obtaining more of the outcome, hence exerting more effort. If the agent has some wealth, the principal will decide, once the agent is receiving a high amount of utility, just to sell the whole firm for whatever wealth the agent has. By moving along the UPF to the right, the principal will sell the firm cheaper, increasing the utility received by the agent.

As the agent gets wealthier, the principal selling the firm to the agent happens sooner in the UPF, that is, for lower values of Δu . In the extreme case when the agent has enough wealth to pay the whole surplus, the principal will be able to write the first best contract and extract the whole surplus.

On the other hand, when the agent has a fixed amount of wealth, increasing his talent will cause the value of the firm, in first and second best outcomes, to increase. This of course is good for both (the share he is receiving increases, and the share of the principal increases as well), however there is a caveat. The more valuable the firm is, the relative wealth, that is the amount of wealth the agent has compared to the value of the firm (ξ), is decreasing, and therefore the agent with more talent is relatively more affected by the CC than a less talented agent. We can observe in Figure 2 how by increasing the level of τ the agent moves from a situation in which he is unaffected by the cash constraint, and therefore a first best outcome is always achieved, to another in which he is considerably cash constrained, up to a point where limited liability rents are created. Let us start with a benchmark result.

Fact 1. *If talent is homogeneous among the agents, and they are wealthy enough such that they are not cash constrained for any firm, then nothing can be said about how firms and agents are going to match.*

Proof. If the poorest agent is rich enough to buy the biggest firm, the principal will always set up a first best contract, and therefore will sell the firm exactly

at its surplus. For the agents, then, all the firms represent the same utility, that is zero, and therefore are indifferent between them. For this reason, any kind of matching can arise. \square

Fact 1 represents the simplest case, in which always first best contracts are written and the principals are able to extract the whole surplus of their firms. Putting this case out of the way, introduce the first proposition of this paper.

Proposition 1. *The economy with principals and agents with moral hazard satisfies generalized increasing differences in (ξ, ω) and (ξ, τ) , which implies that:*

- *For equally talented agents, larger firms will match with wealthier agents.*
- *For equally wealthy agents, larger firms will match with more talented agents.*

Proof. It can be easily shown that ϕ is differentiable in ξ (Lemma 1, Appendix B), the fact that $\partial\phi/\partial\xi$ is non decreasing in τ, ω , and u (Lemma 2, Appendix B), and that ϕ and ψ are type increasing (Lemma 3, Appendix B), Corollary 1 in (Legros and Newman, 2007, p.1097) can be applied, as the requirements for its proof are satisfied, and therefore PAM is obtained for both matches. For a detailed proof please refer to Appendix B. \square

Additional results with discrete types

Wealthier agents will work in bigger firms, and more talented agents will also work in bigger firms. However, what is the equilibrium going to look like? Finding how principals and agents will construct their contracts is not simple, as an equilibrium would imply that everyone is maximizing their expected utility by matching with their partner, and no other principal *steals* the agent of another one. This means not only that the outcome is stable, which is implied by GID, but that the contracts signed by each party can be clearly identified. The conclusions of GID can be applied to a continuum of firms and agents, as well as for a discrete set of them. For simplicity, I will focus in what is left of this article on working with the discrete case, and further I will assume that there are at least as many agents as there are firms.¹⁶

Proposition 2. *Let $\theta = (\omega, \tau)$ and $\theta' = (\omega', \tau')$ be the types of two consecutive agents such that $\omega' \leq \omega$ and $\tau' \leq \tau$. One coordinate (τ or ω) is equal among*

¹⁶This last assumption does not change the matching, but who is keeping the surplus. The assumption that there are more agents than firms seems closer to reality than the opposite situation.

all the agents, whereas the other (ω or τ) is strictly larger. Let $\xi' < \xi$ bet the sizes of two consecutive firms. The equilibrium outcome must satisfy:

- If (θ', ξ') represents the match of the lowest matching types, agent and principal will obtain:

$$\begin{aligned} u^* &= \underline{u}_{\theta'}(\xi') \\ v^* &= \phi(\xi', \theta', u^*) \end{aligned}$$

- Otherwise, let \tilde{v} be the utility of the low type principal. The high type match will obtain:

$$\begin{aligned} u^* &= \max\{\psi(\theta, \xi', \tilde{v}), \underline{u}_{\theta}(\xi)\} \\ v^* &= \phi(\xi, \theta, u^*) \end{aligned}$$

Proof. The first part of the proposition is trivial, as by GID no principal would want to outbid the lowest type principal for the lowest type agent, and therefore there are no incentives for the lowest type principal to provide more utility than the minimum possible, that is $\underline{u}_{\theta'}(\xi')$.

For the second part, assume that $u^* = \psi(\theta, \xi', \tilde{v})$. Note that if the high type principal were to offer less than u^* , say $u^* - 2\epsilon$, then the low type principal could offer $u^* - \epsilon$ and be strictly better off because the UFP is strictly increasing in θ . By offering higher utility to the high type agent, she outbids the high type principal for the high type agent. If $u^* = \underline{u}_{\theta}(\xi)$, the same reasoning applies, as it is sufficient that the high type principal offers enough utility to the agent to avoid the outbidding from the low type principal. \square

In a 2 by 2 world, that is two principals and two or more agents, the low type match will write down a second best contract as if they were in an isolated situation, whereas the high type match will write down a contract that provides the agent at least as much utility as the high type agent would obtain with the low type principal, when this principal is getting his second best utility when matching with the low type agent, that is, when the low type principal is unable to outbid the high type principal for the high type agent.

Proposition 3. *Consider consecutive matches of firms of size $\xi' < \xi$, and agents with types $\theta' < \theta$, such that only ω or τ is equal for all the agents, and for every match, the agents are cash constrained for both firms.*

If the difference in wealth or talent is large enough, i.e. if:

- $\omega - \omega' > \tau \left(2\xi'^4 + \frac{(\xi^2 - \xi'^2)}{4} - \xi'^2 \sqrt{\xi^2 - \xi'^2 + 4\xi'^4} \right)$, when the agents' type is wealth.
- $\frac{\tau'}{\tau} < 2 - \frac{\xi^2}{2\xi'^2}$, when the agents' type is talent.

Then the high type match will write a contract with stronger incentives than the contract it would write out of the market, and therefore closer to the first best output.

Proof. Let $\xi' < \xi$, $\omega' \leq \omega$ and $\tau' \leq \tau$ with only one of these two last inequalities being strict. Let v' represent the utility the low type principal obtains by matching with the low type agent, and assume both agents are cash constrained for both firms. From Proposition 2 we can write the maximum utility the high type agent could get from the low type principal:

$$u = \frac{1}{4} \left(-2(\omega' + \omega) + \xi'^2 \frac{\tau + \Delta\tau}{2} + \xi' \sqrt{\tau(\xi'^2 \Delta\tau + 4\Delta\omega)} \right), \quad (3)$$

where $\Delta\omega = \omega - \omega'$ and $\Delta\tau = \tau - \tau'$. If the contract of the high type match is equivalent to the one they would write out of the market, then the high type agent would receive $\xi^2\tau/8 - \omega$. The conditions in the proposition come from comparing the utility level in (3) against this last expression. \square

Proposition 3 states the conditions such that the competitive pressure imposed by the market makes the high type match write a contract that provides incentives in which the outcome is closer to the first best, compared to the expected outcome in the one-to-one version of the model, reflecting the efficiency brought by competition into this economy. The first part of Proposition 3 says that when the agents' type is wealth, then the more talented they are, and the more different in size are the firms, how much different wealth between high and low type agents needs to be, in order for the competitive pressure to be enough to motivate the high type principal to strengthen the incentives for the high type agent. Note that a bigger firm makes the agent more cash constrained, increasing the limited liability rents, and therefore decreasing the need of more compensation to avoid outbidding from the low type principal (as part of this cost is already covered by the higher limited liability rents). This explains, then, the necessary increase in the wealth of the high type agent compared to the low type to have a contract that provides him more than the limited liability rents.

The second part of Proposition 3 says that when the agents' type is talent, and the size of the smaller firm is less than half of the size of the big one, then for any relationship of talent between the agents, there is no competition enough to make the high type principal to provide extra incentives to the high type agent. However if $\xi' \geq (\sqrt{2}/2)\xi$, then for any pair τ' and τ the contract

written by the high types is going to give the high type agent more limited liability rents than what he would obtain with the high type principal in the absence of market pressure.

The conclusions of Propositions 3 and 4 go in line with the literature, in the sense that market pressure (competitive factors) can increase the difference in expected compensation among the agents more than the difference in the limited liability rents created by the firm size.¹⁷ This would explain why concentrated distributions in talent, for example, can lead to dispersed distributions of compensation as found by Terviö (2008).

Finally, the existence of a minimum wage in this setup could create the situation in which some firms opt to stay out of the market and not operate, unless the pool of agents with such large constraints have very high levels of skills, which would increase the value of the project enough such that $\frac{\xi^2\tau}{4} > -\omega$ again. This goes in line with the fundamental principle that higher wages should go accompanied with improvements in productivity if the firms are to stay functioning.

3 Wealth, Talent and the Matching

I have shown that the economy described in this work satisfies positive assortative matching when the agent's type is either talent or wealth. This suggests that if talent and wealth are positively correlated, that is, more talented agents have higher amounts of wealth, indeed positive assortative matching will arise. However, there are situations in which this does not necessarily happen. In particular, for young agents, their wealth is not correlated with their talent but maybe their cash constraint is influenced by their networks or their family wealth, so the question that remains is: Does positive assortative matching with respect to talent holds for all joint distributions of the agents' talent and wealth?

The corporate finance literature (Terviö, 2008) considered positive assortative matching between agents and principals when considering talent and size as their types. I exploit the model introduced in this article to analyze the impact that wealth can have on the matching between principals and agents. In other words, are there distributions of wealth and talent among agents that could compromise the PAM with respect to talent? Can wealthier but poorly talented agents match with big firms at the same time that talented and poorer agents end up working in small firms?

Proposition 4. *A necessary and sufficient condition to have PAM in talent independently of the distribution of wealth, is that, for any pair of agents, the*

¹⁷Because the agents are risk neutral, this is equivalent to gains in expected utility.

more skilled agent is twice as talented as his less talented counterparts.

Proof. Having $\tau > 2\tau'$ ensures that the curve of the talented agent is always above — i.e. for every level of utility for the agent, the principal gets more profits with the skilled agent than with the rich one. This ensures that the utility possibility frontiers do not cross, and, the normal process to find GID in the 1 dimensional problem holds.

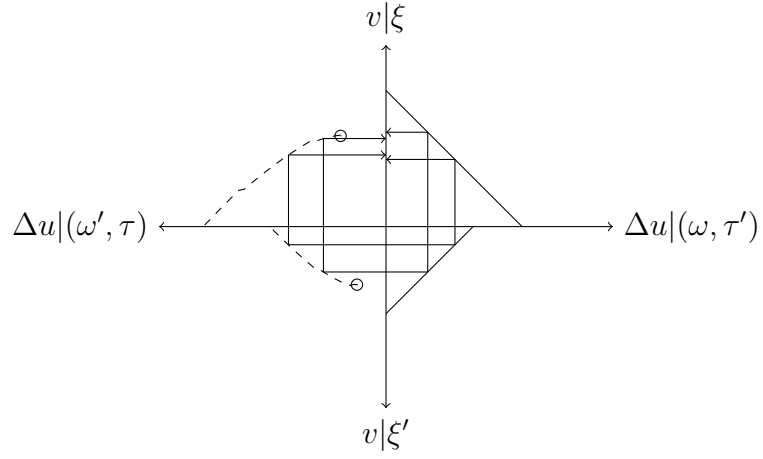
Note that if this condition is not met, GID would be destroyed because the large firm would by the end prefer to keep the low skill high wealth agent. Note that the small firm would bid 0 for the wealthy agent (if condition is not met, the maximum of firm profits is achieved by selling the firm to the wealthy agent). However, this is true for both firms, so the large firm could offer an ϵ of utility to the wealthy agent and overbid for him. The small firm would enter in this loop of bidding for the agent until the utility that can achieve is equal to the maximum profits she can get with both agents is equal, and therefore is indifferent between both agents. It is easy to see that, given that the large firm has the utility possibility frontiers with both agents “above” the small firm, this intersection happens at much higher utility levels for the agent. Therefore, the small firm is indifferent between both, but the large firm still strictly prefers the wealthy agent.

□

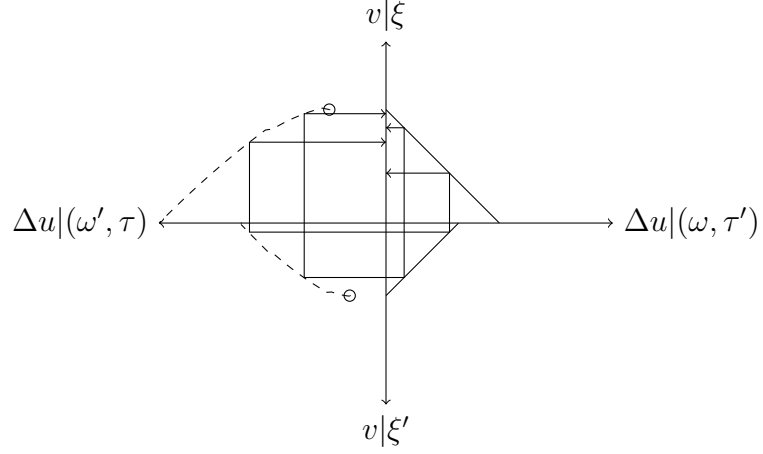
Proposition 4 provides sufficient conditions for the PAM in talent holds, independently of the distribution of wealth among agents. We see exactly the case in in Figure 3, where we observe a graphical representation of the UPF to look for GID or GDD,¹⁸ following Legros and Newman (2007). In detail, we compare four possible matches: In the upper half of each vertical axis we consider the big firm matching with the agents, whereas in the bottom half we consider the small firm matching with the agents. The left half considers the poor but talented agent matching with the firms, whereas the right half considers the wealthier but not so skilled agent matching also with the firms as well. Each axis represents the utilities of each actor, and the curves are the UPF derived from each matching. For simplicity I assume that the poor agent has $\omega = 0$, whereas the rich agent has $\omega > 1$, which ensures that he is cash unconstrained with any firm.

We observe (Figure 3a) that when having a poor but skilled agent against a rich but less capable agent we might no longer have PAM with respect to talent and firm size. This happens when the difference in their talent is below some threshold ($\tau - \tau' < \underline{\tau}$). Furthermore, if we start assuming a level \bar{v}

¹⁸LN define generalized decreasing differences (GDD) as a sufficient condition for negative assortative matching (NAM), where the lower type principal can outbid the high type principal for the high type agent.



(a) $\tau = 0.4, \tau' = 0.3$.



(b) $\tau = 0.3, \tau' = 0.15$.

Figure 3: Assessment of the bidimensional matching between agents and principals.

as reservation profits for the firms, then we can even end up with negative assortative matching when considering firm size and agent talent, if rich agents have poor skills and poor agents are talented. Conversely, if the difference in talent between the two agents is sufficiently high (Figure 3b), then the effect of the limited liability becomes irrelevant, and positive assortative matching between talent and firm size arises.

Note that where the UPF intersects the horizontal axis (the agent's change in utility) only changes with τ and ξ , whereas ω is crucial to determine where the UPF reaches the maximum value for ϕ (the principal's utility). Because of this, if we analyze the matching between both of the agents, (ω, τ') and (ω', τ) , with a firm of a given size, then the poorer agent will always have the bottom of his UPF to the right of the UPF generated by the matching of the wealthier one. However, if the difference in talent is small, the poorer agent's UPF will reach a lower maximum (the isolated second best contract

outcome) on the vertical axis than the UPF of the wealthier agent (the first best outcome, of a slightly smaller outcome because of the lower talent). This crossing of the UPFs is critical, as the firms prefer one agent or the other, depending on how much utility they will have to provide to the agent in order to make the matching stable. The UPFs will cross if and only if the difference in talent is low enough.

4 Conclusions

The model developed here allows to understand some implications of the traditional moral hazard framework on the matching between principals and agents. In particular I focus on the effect of the agent's wealth on his relationship with firms of different size. I show that, with risk neutral principals and agents, wealth makes the agent cash constrained for a lower number of firms (the smaller ones), and further, if he is not cash constrained, his limited liability rents decrease with his wealth, and increasingly so in the firm's size. Another result is that the size of the firm, measured in earnings, increases both agent and principal's utility and compensation. Therefore a first conclusion is that the agent would prefer to work in big firms, for which he would be cash constrained and therefore able to extract limited liability rents. On the other hand, firms prefer to hire wealthier agents, as this would allow them to reduce the limited liability rents the agent can extract from the surplus. This is true only as long as the agent is cash constrained. This will happen for higher levels of wealth, the higher size of the firm, or conversely, for a given size of the firm, for lower levels of wealth. If the firm is small, the principal can do just fine with poorer agents, as less wealth from their side is required to keep them from being cash constrained.

In order to tackle the question of how principals and agents match, I adapt the techniques developed in Legros and Newman (2007) and use generalized increasing differences to obtain endogenous positive assortative matching between principals and agents when considering the firm's size and the agent's wealth or talent. I also describe conditions on the parameter space to describe the contract associated to each match, and its efficiency. In particular, I find that when types are closer, the market pressure is higher and makes the high type match to set up a contract that creates a higher surplus than that which could be obtained in a 1 to 1 situation. I also find that the lowest type will always write a contract that is equivalent to an outside the market outcome, when principals retain the bargaining power.

Finally, I provide an example in which a poor agent with high talent, and a wealthy agent with poor talent, match with firms of different size. I find that if the difference in talent is small, then there will be no positive assortative

matching with respect to talent and firm size. However, if the difference between the agents' talent is sufficiently high, PAM is maintained. Considering that wealth is not necessarily perfectly correlated with talent in a sample with similar agents, this can be an issue for empirical considerations. In particular, given that Akerberg and Botticini (2002) has already shown the negative impact that neglecting the matching considerations can have on empirical results, I believe this paper provides some advances for the matching and the contracts following the question raised in Edmans et al. (2009) regarding the effect of wealth on incentives, and as pointed out by them and others since Akerberg and Botticini (2002), that means necessarily understanding the effect of wealth on the principal-agent matching.

This paper provides a parsimonious framework that incorporate the effect limited liability could have on the competitive assignment of agents to firms when these agents differ in talent *and* wealth, while firms have different sizes, and taking into consideration that utility is not perfectly transferable between the parts. The empirical literature has found (and often assumed) that positive assortative matching arises when considering firm size and agent talent. I show that the effect limited liability channel cannot be taken for granted, and therefore researchers must be careful, in particular when assuming that talent is homogeneous between agents (Terviö, 2008). As pointed out by Terviö (2008) “.. *the matching of CEOs with exogenously heterogeneous firms has a genuinely important role in driving the competitive levels of CEO pay...*”, and this manuscript aims to help providing insights in that area.

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Appendix

A Positive Assortative Matching as in Legros and Newman (2007)

As shown in Legros and Newman (2007), consider two principals of types p, p' with $p' < p$, and two agents types a, a' with $a' < a$. Consider further than when matched, the utility possibility frontier (UPF) is defined by $\phi(p, a, v)$ where $\phi(p, a, v)$ represents the utility obtained by the principal type p when matched with the agent type a , and this agent obtains utility v . In the same fashion, let $\psi(a, p, u)$ represent the utility of an agent type a when matched with a principal type p when this principal is getting utility u . It happens that both ψ and ϕ represent the same UPF from a different reference point.

Legros and Newman (2007) claim that positive assortative matching (PAM), for $u \in [0, \phi(p', a, 0)]$ arises when :

$$\phi(p, a, \psi(a, p', u)) \geq \phi(p, a', \psi(a', p', u)) \quad (4)$$

$u \in [0, \phi(p', a, 0)]$ means that u can take any value between zero and the maximum utility the low type principal (p') can obtain with the higher type agent (a). This is so, because the upper bound of the interval corresponds to when all the surplus remains with the principal (so the agent's utility is zero).

Now $\psi(a, p', u)$ is the maximum utility the low type principal (p') can give the high type agent (a) if she desires to get utility u . In the same way, $\psi(a', p', u)$ is the maximum utility the low type principal (p') can give the low type agent (a') if she desires to get that same amount of utility u .

Finally, $\phi(p, a, \psi(a, p', u))$ is the utility the high type principal (p) can get when matched with the high type agent (a), while giving this agent utility $\psi(a, p', u)$, that in turns is the maximum utility the low type principal (p') can give this agent (a) if she desires to obtain utility u . This utility — $\phi(p, a, \psi(a, p', u))$ — must be larger than $\phi(p, a', \psi(a', p', u))$ that represents the utility the high type principal (p) would get by matching with the low type agent (a') and giving him utility $\psi(a', p', u)$, that in turns is the maximum utility the low type principal (p') could give the low type agent (a') when she wants to obtain utility u .

In words, for any utility level the low type principal (p') would want to achieve if matching with the higher type agent (a), the high type principal (p) would be always willing to provide at least that same amount of utility to that agent (a) instead of being resigned to match with the low type agent (a'), as by outbidding the low type principal (p') for that high type agent (a)

she achieves always higher (or equal) utility levels.

B Proof of Proposition 1

In order to look for GID or PAM in this model, I will refer to Corollary 1 in LN, where they use the assumption that ϕ is twice differentiable to obtain PAM, by looking at the signs of its second derivatives. There is a caveat though: in our model ϕ is not twice differentiable. Later on, in this appendix, I discuss that for LN's corollary, it is enough for PAM that ϕ is differentiable in ξ , and that this derivative is increasing in u and the agent's type. These conditions establish that the gains of a principal by matching with a higher type counterpart are greater (a supermodularity condition) than when matching with lower type counterparts, plus that for higher values of utility given to the agent along the UPF, these gains also increase. This will ensure that the high type principal can outbid the low type principal for the high type agent.

Lemma 1. *The UPF described by $\phi(\xi, \theta, u)$ is:*

- *Continuous and strictly decreasing in u for $u \in U_\xi(\theta)$.*
- *Differentiable in u for $u \in \text{int}(U_\xi(\theta))$.*
- *Differentiable in ξ .*

Proof. To prove continuity it is enough to verify that $\phi(\xi, \theta, \xi^2\tau/8 - \omega) = \omega + \xi^2\tau/4$ and that $\phi(\xi, \theta, \xi^2\tau/2 - \omega) = \omega$. Both are verified using simple algebra.

Every piece of the UPF is differentiable in u . Therefore, it is sufficient to verify that the derivatives coincide when the function changes its functional form. At the first best surplus (and therefore in that section of the UPF) the UPF has slope -1 . The derivative of the UPF when PC and CC are binding is $(\xi\sqrt{2\tau(u+\omega)})/(2(u+\omega)) - 2$. After replacing $u = \xi^2\tau/2 - \omega$ we obtain -1 , and then the UPF is differentiable in the interior of $U_\xi(\theta)$. The part of the UPF in which only the CC is binding is a point, and lies in the minimum value of utility the agent can get from the matching, and therefore does not belong to the interior of $U_\xi(\theta)$.

Finally, as the function is continuous and every piece of it is strictly decreasing in u , the UPF is decreasing in u .

Now analyzing $\partial\phi/\partial\xi$, we need to study again its intervals. ϕ written as a function of ξ is:

$$\phi(\xi, \theta, \bar{u}) = \begin{cases} \frac{\xi^2 \tau}{2} - \bar{u} & \text{if } \xi \leq \sqrt{\frac{2(\bar{u} + \omega)}{\tau}} \\ \xi \sqrt{(\bar{u} + \omega) 2\tau} - 2\bar{u} - \omega & \text{if } \sqrt{\frac{2(\bar{u} + \omega)}{\tau}} < \xi \leq \sqrt{\frac{8(\bar{u} + \omega)}{\tau}} \\ \frac{\xi^2 \tau}{4} + \omega & \text{if } \xi > \sqrt{\frac{8(\bar{u} + \omega)}{\tau}} \end{cases}$$

Using simple algebra it can be shown that this function is continuous in ξ . Moreover, taking the derivatives in each piece, and replacing the boundaries of each piece of the function, it can also be verified to be differentiable in ξ . \square

Lemma 1 is instrumental in the construction of the proof of the final proposition. It says that the UPF is well behaved and follows a standard principle: The more utility is given to the agent, the less utility the principal will obtain.

Lemma 2. $\partial\phi/\partial\xi$ is continuous and increasing in τ , ω , and u .

For the proof of Lemma 2 please refer to Appendix B. This is the first stone to build up the supermodularity type of characteristics for ϕ in order to obtain PAM. Further, it is necessary that ϕ does not decrease in the agent's type, nor ψ decrease in the principal's type. This is formalized in the following lemma,

Lemma 3. $\phi(\xi, \theta, u)$ and $\psi(\theta, \xi, v)$ are type increasing, that is:

- ϕ is non decreasing in ω and τ .
- ψ is non decreasing in ξ .

Proof. From Table 1 it can be seen that, at least piece-wise, both ϕ and ψ are non decreasing in ω or τ the former, and ξ the latter. Continuity of ϕ and ψ in each of the relevant variables, which is easily shown, is sufficient then to obtain the type increasing property. \square

Lemma 3 shows that the surplus increases when increasing ω , τ , or ξ , formalizing what can be observed in Figures 1, 2, and 4 in which the UPF shifts upward when increasing any of those parameters. This implies that for any level of u that is feasible, the principal obtains at least as much utility with a richer, or more talented agent.

B.1 About Corollary 1 in Legros and Newman (2007)

In this Appendix I explore if the conditions which are sufficient to have generalized increasing differences (GID) are satisfied by the principal-agent model

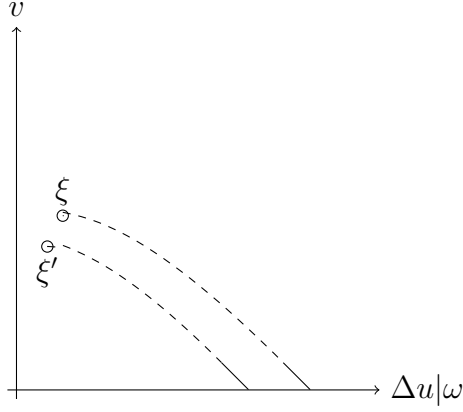


Figure 4: $\xi' < \xi$, $\omega \in [0, \frac{\xi^2\tau}{8}]$ with $\tilde{\xi} \in \{\xi', \xi\}$

developed in the main text. The utility possibility frontier (UPF) generated by the original model is found in Table 1.

In this UPF $E[v]$ represents the utility obtained from the principal after signing the contract with the agent. Each column represents the situation in which the agent is cash constrained, but the participation constraint is not binding, when the cash constraint and the participation constraint are both binding, and finally when only the participation constraint is binding. The relation between the variables that define in what situation we are, is given by the base wage determined when the agent is not cash constrained, against his wealth. If the fixed part of the optimal wage, assume a cash-unconstrained agent, is lower than his minus wealth, then the agent is not cash constrained. Then, the agent is cash constrained if:

$$\bar{u} - \frac{\xi^2\tau}{2} < -\omega$$

This implies that the agent is not cash constrained if $\xi \geq \sqrt{\frac{2(\bar{u}+\omega)}{\tau}}$, so $\phi(\xi, \theta, \bar{u}) = \frac{\xi^2\tau}{2} - \bar{u}$ if $\xi \geq \sqrt{\frac{2(\bar{u}+\omega)}{\tau}}$. Note that this condition is equivalent to $\bar{u} \leq \frac{\xi^2\tau}{2} - \omega$ when looked from the point of view of \bar{u} .

With that, $\phi()$ can be written as:

$$\phi(\xi, \theta, \bar{u}) = \begin{cases} \frac{\xi^2\tau}{2} - \bar{u} & \text{if } \xi \leq \sqrt{\frac{2(\bar{u}+\omega)}{\tau}} \\ \xi\sqrt{(\bar{u}+\omega)2\tau} - 2\bar{u} - \omega & \text{if } \sqrt{\frac{2(\bar{u}+\omega)}{\tau}} < \xi \leq \sqrt{\frac{8(\bar{u}+\omega)}{\tau}} \\ \frac{\xi^2\tau}{4} + \omega & \text{if } \xi > \sqrt{\frac{8(\bar{u}+\omega)}{\tau}} \end{cases}$$

Which is above written as a function of ξ . This function is continuous in ξ . In order to be differentiable, let's look at its derivatives:

$$\frac{\partial \phi(\xi, \theta, \bar{u})}{\partial \xi} = \begin{cases} \xi\tau & \text{if } \xi \leq \sqrt{\frac{2(\bar{u}+\omega)}{\tau}} \\ \sqrt{(\bar{u}+\omega)2\tau} & \text{if } \sqrt{\frac{2(\bar{u}+\omega)}{\tau}} < \xi \leq \sqrt{\frac{8(\bar{u}+\omega)}{\tau}} \\ \frac{\xi\tau}{2} & \text{if } \xi > \sqrt{\frac{8(\bar{u}+\omega)}{\tau}} \end{cases}$$

The derivatives coincide in each interval, and therefore ϕ is differentiable in ξ . It remains to determine if $\partial\phi/\partial\xi$ is differentiable in \bar{u} , ω , and τ .

B.1.1 $\partial\phi/\partial\xi$ for $(\bar{u} + \omega)$

$$\frac{\partial \phi(\xi, \theta, \bar{u})}{\partial \xi} = \begin{cases} \frac{\xi\tau}{2} & \text{if } \bar{u} + \omega < \frac{\xi^2\tau}{8} \\ \sqrt{(\bar{u} + \omega)2\tau} & \text{if } \frac{\xi^2\tau}{8} \leq \bar{u} + \omega < \frac{\xi^2\tau}{2} \\ \xi\tau & \text{if } \bar{u} + \omega \geq \frac{\xi^2\tau}{2} \end{cases}$$

Which is continuous in $\bar{u} + \omega$. Derivatives with respect to $\bar{u} + \omega$ in each interval are 0, $\frac{\tau}{\sqrt{2}\sqrt{\tau(\bar{u}+\omega)}}$, and 0 respectively. Evaluating in the extremes of the interval it gives $\frac{2}{\xi}$ and $\frac{1}{\xi}$ respectively. Therefore it is not differentiable in $\bar{u} + \omega$.

However, it is necessary that $\partial\phi/\partial\xi$ is increasing in \bar{u} which it is. As the function is continuous, and increasing in \bar{u} inside each interval, then $\partial\phi/\partial\xi$ is **increasing in $\bar{u} + \omega$** , which implies that it is increasing in \bar{u} and ω .

B.1.2 $\partial\phi/\partial\xi$ for τ

$$\frac{\partial \phi(\xi, \theta, \bar{u})}{\partial \xi} = \begin{cases} \xi\tau & \text{if } \tau \leq \frac{2(\bar{u}+\omega)}{\xi^2} \\ \sqrt{(\bar{u} + \omega)2\tau} & \text{if } \frac{2(\bar{u}+\omega)}{\xi^2} < \tau \leq \frac{8(\bar{u}+\omega)}{\xi^2} \\ \frac{\xi\tau}{2} & \text{if } \tau > \frac{8(\bar{u}+\omega)}{\xi^2} \end{cases}$$

Which is continuous in τ . The derivatives with respect to τ are: ξ , $\frac{\bar{u}+\omega}{\sqrt{2\tau(\bar{u}+\omega)}}$, and $\frac{\xi}{\tau}$ respectively. All of them positive, and therefore $\partial\phi/\partial\xi$ is increasing in τ in each interval. This added to continuity gives that $\partial\phi/\partial\xi$ is **increasing in τ** .

B.1.3 GID

The sufficiency conditions expressed in corollary 1 in Legros and Newman (2007, p. 1083) are:

- For GID in ξ and ω ,

$$\frac{\partial^2 \phi(\xi, \theta, u)}{\partial \xi \partial \omega} \geq 0 \quad \text{and} \quad \frac{\partial^2 \phi(\xi, \theta, u)}{\partial \xi \partial u} \geq 0$$

- For GID in ξ and τ ,

$$\frac{\partial^2 \phi(\xi, \theta, u)}{\partial \xi \partial \tau} \geq 0 \quad \text{and} \quad \frac{\partial^2 \phi(\xi, \theta, u)}{\partial \xi \partial u} \geq 0$$

In their proof, they use $\partial^2 \phi / \partial \xi \partial \omega$ or $\partial^2 \phi / \partial \xi \partial \tau$ to obtain that the derivative of ϕ with respect to ξ is increasing in the agent's type and utility (Legros and Newman, 2007, p.1097). Even though this function is not twice differentiable in the model presented here, we have shown that it is increasing in all the necessary variables and therefore the model in this economy satisfies GID in ξ, ω and ξ, τ .

B.2 Derivation of the Partial Model

In this Appendix I provide the detailed steps to arrive from the maximization problem of the principal in (1) to the results shown in Table 1.

Given the simplifying assumptions in the model, it is possible to solve the problem by using the first order approach (as the agent's problem has a unique solution),¹⁹ the optimal e for the agent, given a pair of a and b is given by the following condition:

$$\begin{aligned} p'(e)b &= c'(e) \quad \text{or,} \\ b &= \frac{c'(e)}{p'(e)} \end{aligned} \tag{5}$$

Equation (5) implies that $e = b\tau$. As expected, the amount of effort exerted by the agent is increasing in the distance between the wage in the good and bad state (in other words, the size of the bonus), as well as in the level of his ability. We can replace then the first order condition (5) into the other equations of the principal's problem eliminating the variable b from it. The new problem for the principal is:

$$\begin{aligned} \max_{e,a} \quad & -a + e[\xi - c'(e)] & (6) \\ \text{s.t.} \quad & a + ec'(e) - c(e) \geq \bar{u} & (\text{PC.2}) \\ & a \geq -\omega & (\text{CC.2}) \end{aligned}$$

This reduced problem has a particular advantage. Besides having less variables to consider, it allow us to put the attention in the cash constraint for the agent. In particular we are interested on how affected are the incentives when

¹⁹Grossman and Hart (1983) show how, under certain conditions, satisfied in this model, it can be solved in two stages, first the agent's problem, given the wages, and later the principal's problem.

(CC.2) is relevant for the principal's optimization. This will be the case as long as (CC.2) is binding. We will proceed assuming that the agent is not cash constrained, and solve the optimal compensation scheme. This is equivalent to think that ω is big enough such that the a obtained by maximizing subject to (PC.2) is higher than $-\omega$. Later we will solve the problem considering the opposite case, to finalize with the case in which both are binding.

Let ω be such that (CC.2) is not binding. From the new participation constraint we obtain the minimum a that would make the agent sign the contract. This a is given by:

$$a = \bar{u} + c(e) - ec'(e) = \bar{u} - c(e)$$

The optimal a depends positively on the agent's reservation utility but negatively on effort. This is the most direct way in which we can observe how the cash constraint (that forbids at some point decreasing a no matter the level of e) impedes the principal to achieve an efficient outcome. The optimal level for a can be replaced in the objective function for the principal's maximization problem described in (6) to end up with the following problem:

$$\max_e -\bar{u} + e\xi - c(e)$$

Whose first order condition with respect to e is $\xi = c'(e)$. This implies that the marginal benefit of e should be equal to its marginal cost, which is exactly the optimality condition in a first best situation. The solution should satisfy $e^* = \tau\xi$. It should not be surprising to find that for higher values of the output in the good state, the higher the contracted effort, as it happens with the agent's talent.

The wages set by the principal are $a = \bar{u} - \frac{\xi^2\tau}{2}$ and $b = \xi$. As stated previously, the principal charges the expected surplus of the operation whereas letting the agent keep the whole good outcome of the firm.

The expected utility for agent and principal are $\bar{u} + \omega$ and $\frac{\xi^2\tau}{2} - \bar{u}$ ($= -a$) respectively. This shows also that the agent, when not cash constrained, is unable to extract any limited liability rents from the principal. This result is widely known in the moral hazard literature. The principal therefore keeps the whole surplus of the project, and she only needs to satisfy the agent's reservation utility. This is equivalent to a first best outcome, when the principal has full knowledge of the agent's actions.

To obtain these results, it is important to recall that we have assumed that equation (CC.2) is not binding. Having an expression for a from the participation constraint (CC.2), we can find a condition that indicates if either

the participation constraint (PC.2) or the cash constraint (CC.2) is binding. For equation (CC.2) not to be binding it must hold that:

$$\bar{u} - \frac{\xi^2 \tau}{2} > -\omega \quad (7)$$

From equation (7) we obtain already some conclusions. The higher the agent's reservation utility (\bar{u}) and the agent's wealth (ω), the less likely the cash constraint (CC) is going to be binding. In the opposite direction, the larger the size of the firm (ξ), the more likely the cash constraint is to be binding.

Assume now that equation (CC.2) is binding. That implies immediately that $a = -\omega$, and given that equation (IC) hasn't changed, $b = c'(e)$. Replacing that in the objective function, now the principal optimizes:

$$\max_e -(-\omega) + e[\xi - c'(e)]$$

That yields as solution $e^* = \frac{\tau \xi}{2}$. It is direct to see how the optimal effort has diminished. The optimal compensation scheme is now given by $a = -\omega$ and $b = \frac{\xi}{2}$. The expected utility for the agent and the principal are $\frac{\xi^2 \tau}{8}$ and $\omega + \frac{\xi^2 \tau}{4}$ respectively.

An interesting result is that the agent's expected utility is decreasing in ω and increasing in ξ . Remember that the higher ω , the less possibilities has the agent to extract limited liability rents from the principal. Looking to the principal, v is increasing in both ω and ξ .

Finally there is the situation in which both constraints PC and CC are binding (IC must always be binding with asymmetric information). In this case, we determine the optimal level of effort without having to maximize the principal's utility, as this level is determined by the system of equations given by the *PC*, *CC* and *IC*. We obtain that the implemented level of effort is:

$$e = \sqrt{(\bar{u} + \omega)2\tau} \quad (8)$$

Equation (8) implies that e is increasing in ω , τ , and \bar{u} . As the participation constraint is binding, the agent is still getting the expected utility \bar{u} . However, as $a = -\omega$, a is decreasing in the agent's wealth, and therefore the effectively paid bonus should increase. As it can be observed, to implement higher levels of effort it is necessary to have higher values of b as well. As consequence the expected bonus increases in ω , compensating the agent for the decrease in his fixed pay a and keeping his utility at \bar{u} . Conversely, decreasing the agent's wealth implies a higher fixed wage a , and therefore for the agent to

obtain his reservation utility, a lower bonus is required implementing a lower level of effort. The principal suffers by having to implement lower levels of effort (compared to first best) and paying a higher fixed compensation, decreasing her expected utility (as she is getting lower probability of receiving ξ). The principal's expected utility can be written as:

$$E[v|\text{CC and PC binding}] = \xi\sqrt{(\bar{u} + \omega)2\tau} - 2\bar{u} - \omega$$

It is very important to recall that when the CC is binding for the agent, the PC must be satisfied, so the agent is always getting \bar{u} or more. This implies that the agent is always at least as good (always weakly better) when he is cash constrained. On the opposite side, the principal is getting all the surplus when the agent is not cash constrained.