

Producer Theory

Lecture 10: The Geometry of Costs — Short Run vs Long Run

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Welcome Back! 🙌

We've crossed the aisle:

 Lectures 5–9: **Consumer Theory** (demand side)

 Lectures 10–15: **Producer Theory** (supply side)


 **Today's Goal:** Understand the **cost structure** of firms

How do costs behave as production changes?

From Consumer to Producer

The Other Side of the Market


Consumer (Lectures 5–9):

- Maximizes **utility** 
- Subject to **budget constraint**
- Generates **demand curves**
- Tools: indifference curves, MRS, budget line

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 **Key parallel:** Just as consumers face trade-offs with limited income, producers face trade-offs with limited resources and technology.

Producer (Lectures 10–15):

- Maximizes **profit** 
- Subject to **cost structure**
- Generates **supply curves**
- Tools: cost curves, marginal analysis

The Producer's Problem

THE PRODUCER'S GOAL

A firm wants to **maximize profit**. To do so, it must understand its costs at every level of output.

$$\pi = \text{Total Revenue} - \text{Total Cost} = TR - TC$$

Before we can find the profit-maximizing quantity (next lecture), we need to master costs.

Today we answer:

- **1** What types of costs does a firm face?
- **2** How do costs change as output increases?
- **3** What is the difference between short run and long run?

Short Run vs Long Run

Defining the Time Horizons



SHORT RUN VS LONG RUN

Short run: A period in which **at least one** factor of production is **fixed** (cannot be changed). Typically, capital (machines, buildings) is fixed.

Long run: A period long enough that **all** factors of production are **variable**. The firm can adjust everything: equipment, factory size, workforce.

 **Short Run** — Tourism example:

A hotel has 100 rooms. It **cannot build more rooms** this season. It can only hire/fire staff (variable input).

 **Long Run** — Tourism example:

The hotel chain can build a **new wing**, open in a **new city**, or **exit** the market entirely.

Inputs: Fixed vs Variable



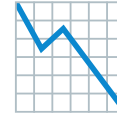
| | | |
|----------------------|---|--------------------------------|
| Definition | Cannot change in the short run | Can be adjusted at any time |
| Examples | Building, machines, lease contracts | Workers, raw materials, energy |
| Tourism | Hotel building, aircraft, kitchen equipment | Staff, food supplies, fuel |
| Cost behavior | Same cost regardless of output | Cost changes with output |

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THE KEY DISTINCTION

In the **short run**, the firm has both fixed and variable inputs. In the **long run**, **all** inputs are variable — there are no fixed costs.

The Law of Diminishing Returns



LAW OF DIMINISHING RETURNS

When we keep adding more of a **variable input** (e.g., workers) to a **fixed input** (e.g., a kitchen), eventually each additional worker adds **less and less** to total output.

Tourism example: A restaurant kitchen with 5 stoves 🔍

| Extra chefs hired | Extra meals/hour produced | Phase |
|-------------------|---------------------------|--|
| 1st chef | +10 meals | Initial Setup |
| 2nd chef | +25 meals | Increasing returns (Specialization) |
| 3rd chef | +18 meals | Diminishing returns (Space starts to limit) |
| 4th chef | +10 meals | Further diminishing |
| 5th chef | +3 meals | Crowding (Stoves are fully occupied!) |

💡 **Why?** The fixed input (kitchen space) gets **congested**. Chefs bump into each other, wait for stoves, compete for prep space.

Types of Costs in the Short Run

The Cost Family

All short-run costs derive from three building blocks:


Fixed Cost (FC)

Costs that **do not change** with output.

Must be paid even if $Q = 0$.

Hotel: mortgage, insurance, property tax

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 **Important:** FC is a **sunk cost** in the short run — it cannot be recovered. Remember Lecture 3!

Variable Cost (VC)

Costs that **change** with the level of output.

Zero when $Q = 0$.

Hotel: staff wages, laundry, food, energy *Hotel:* everything combined

+ Total Cost (TC)

The sum of all costs.

$$TC = FC + VC$$

A Worked Example: Algarve Beach Bar

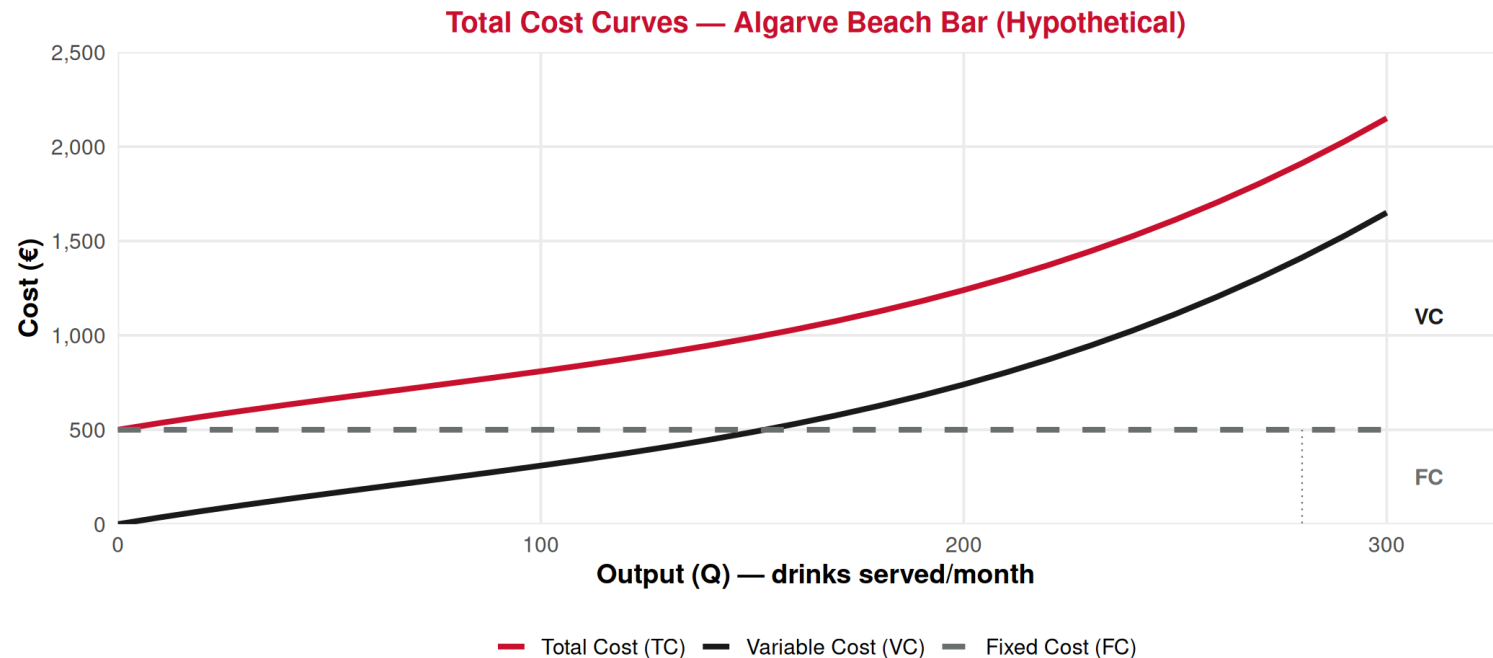
Imagine a small beach bar in Albufeira. The owner pays **€500/month** in rent (fixed), and hires staff and buys supplies (variable).

| Output (Q) | Fixed Cost (FC) | Variable Cost (VC) | Total Cost (TC = FC + VC) |
|------------|-----------------|--------------------|---------------------------|
| 0 | €500 | €0 | €500 |
| 50 | €500 | €200 | €700 |
| 100 | €500 | €350 | €850 |
| 150 | €500 | €550 | €1,050 |
| 200 | €500 | €800 | €1,300 |
| 250 | €500 | €1,150 | €1,650 |
| 300 | €500 | €1,650 | €2,150 |

Output = drinks served per month (hypothetical illustrative example)

👉 Notice: **FC stays at €500** always. **VC increases** — and it increases **faster and faster** (diminishing returns to the staff in the small bar!).

Plotting the Total Cost Curves



👉 **TC and VC have the same shape** — TC is just VC shifted **up by FC**. The vertical gap between TC and VC is always exactly **€500** (the fixed cost).

Average and Marginal Costs

Why Averages Matter 🤔

Total costs tell us the **whole bill**, but firms need to know the **cost per unit** to set prices and make decisions.

AVERAGE COSTS

$$AFC = \frac{FC}{Q} \quad AVC = \frac{VC}{Q} \quad ATC = \frac{TC}{Q} = AFC + AVC$$

Average Fixed Cost (AFC):

- FC spread over more units
- **Always falls** as Q increases
- 🏠 A hotel's mortgage cost per guest falls as occupancy rises

Average Variable Cost (AVC):

- Variable cost per unit
- Falls first, then **rises** (due to diminishing returns)
- 🍴 Food cost per meal — efficient at moderate volume, rising when kitchen is overloaded

Marginal Cost: The Most Important Curve

MARGINAL COST (MC)

The **additional cost** of producing **one more unit** of output.

$$MC = \frac{\Delta TC}{\Delta Q} = \frac{\text{Change in Total Cost}}{\text{Change in Output}}$$

Equivalently: $MC = \frac{\Delta VC}{\Delta Q}$ (since FC doesn't change!)

Intuition: If producing 100 drinks costs €850 total and producing 101 drinks costs €856, then $MC = €6$.

Why is MC so important?

- It tells the firm whether producing one more unit **adds to or subtracts from** profit
- Rule of thumb: **produce more if the price you receive > MC**
- MC is the **foundation of the supply curve** (Lectures 12–14)

Computing Averages and MC: Beach Bar

| Q | FC | VC | TC | AFC = FC/Q | AVC = VC/Q | ATC = TC/Q | MC = $\Delta TC/\Delta Q$ |
|-----|-----|-------|-------|------------|------------|------------|---------------------------|
| 0 | 500 | 0 | 500 | — | — | — | — |
| 50 | 500 | 200 | 700 | 10.00 | 4.00 | 14.00 | 4.00 |
| 100 | 500 | 350 | 850 | 5.00 | 3.50 | 8.50 | 3.00 |
| 150 | 500 | 550 | 1,050 | 3.33 | 3.67 | 7.00 | 4.00 |
| 200 | 500 | 800 | 1,300 | 2.50 | 4.00 | 6.50 | 5.00 |
| 250 | 500 | 1,150 | 1,650 | 2.00 | 4.60 | 6.60 | 7.00 |
| 300 | 500 | 1,650 | 2,150 | 1.67 | 5.50 | 7.17 | 10.00 |

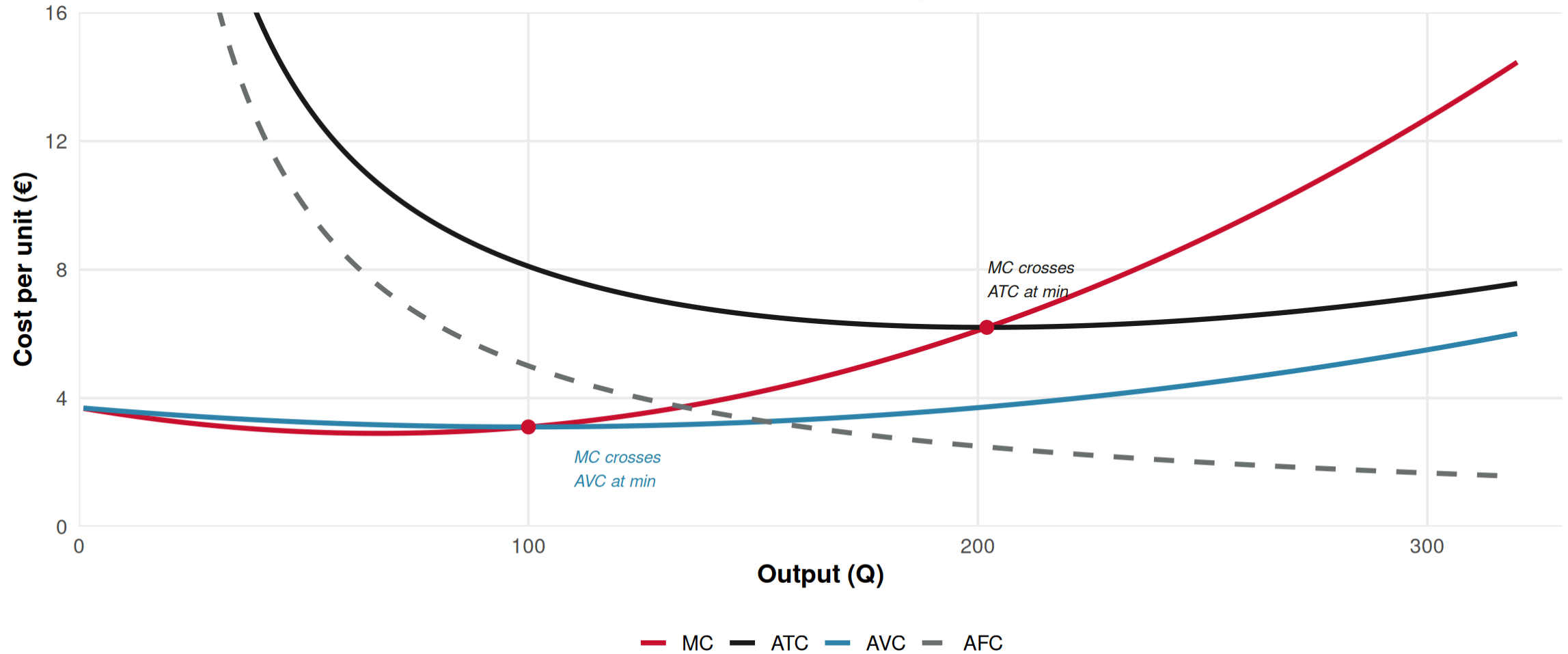
Hypothetical illustrative example

👉 **Key observations:**

- **AFC** always falls (spreading the fixed cost)
- **AVC** falls then rises (U-shaped) — minimum at Q = 100 (AVC = €3.50)
- **ATC** falls then rises (U-shaped) — minimum at Q = 200 (ATC = €6.50)
- **MC** falls then rises — and **crosses AVC and ATC at their minimums!**

The Geometry of Costs

Average and Marginal Cost Curves (Hypothetical Beach Bar)





Why Does MC Cross AVC and ATC at Their Minimums?

Think of it like your **grade point average** 🎓

The “exam grade” analogy:

- Your **average grade** = ATC (or AVC)
- Your **next exam grade** = MC

If your next exam grade is **below** your average → your average **falls** ↓

If your next exam grade is **above** your average → your average **rises** ↑

If your next exam grade **equals** your average → average **stays the same** (minimum!)

Applied to costs:

- When $MC < AVC$ → AVC is **falling** ↓
- When $MC > AVC$ → AVC is **rising** ↑
- When $MC = AVC$ → AVC is at its **minimum**

Same logic applies to ATC!

👉 **MC always crosses average curves at their lowest point.**

This is a **mathematical fact**, not an economic assumption.

The Shapes You Must Know

Total Cost Curves:

- **FC**: Horizontal line (constant)
- **VC**: Starts at origin, S-shaped (first rises slowly, then steeply)
- **TC**: VC shifted up by FC (same S-shape)

The S-shape comes from diminishing returns — costs eventually accelerate.

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 **Don't confuse**: MC can be **below** ATC while ATC is still **falling**. ATC rises only after MC **crosses above** it.

Per-Unit Cost Curves:







- **AFC**: Always declining (hyperbola)
- **AVC**: U-shaped (falls then rises)
- **ATC**: U-shaped, above AVC (the gap is AFC)
- **MC**: U-shaped, crosses AVC and ATC at their minimums

 As Q grows, ATC gets closer to AVC because AFC shrinks.

Tourism Application

Costs in the Hotel Industry

Fixed costs (don't change with occupancy):






-  Mortgage / building lease
-  Insurance
-  Property tax
-  Booking system software license
-  Core management salaries
-  Scheduled maintenance

These costs explain why hotels hate **empty rooms** — FC is already committed!

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 The difference between FC and VC explains why **low-season pricing** exists: covering variable costs and **some** fixed costs is better than covering **none**.

Variable costs (change with occupancy):



-  Cleaning and laundry per room
-  Energy and water per guest
-  Breakfast/meal supplies
-  Linen and amenity replacement
-  Part-time/seasonal staff

That's why hotels offer last-minute discounts — as long as $P > MC$ of hosting one more guest, it's worth it!

Spreading Fixed Costs: Why Airlines Overbook

The airline cost structure (illustrative):

An Algarve charter flight has roughly:

-  **High fixed costs:** aircraft lease, crew salaries, airport fees, fuel (mostly fixed per flight)
-  **Low variable costs per passenger:** meals, baggage handling, marginal fuel

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SPREADING THE FIXED COST

If a flight costs €20,000 to operate (mostly fixed) and has 200 seats:

- **100 passengers:** AFC = €200/passenger
- **150 passengers:** AFC = €133/passenger
- **200 passengers:** AFC = €100/passenger

Every additional passenger **dramatically lowers** the cost per passenger!

👉 This is why airlines practice **overbooking** and **dynamic pricing** — filling seats spreads the enormous fixed cost.

Long Run Costs

In the Long Run, Everything is Variable

In the **long run**, the firm can adjust **all** inputs — including building size, number of machines, and technology.

LONG-RUN AVERAGE COST (LRAC)

The lowest possible average cost for each level of output, when the firm is free to choose the **optimal scale** (size) of operations.

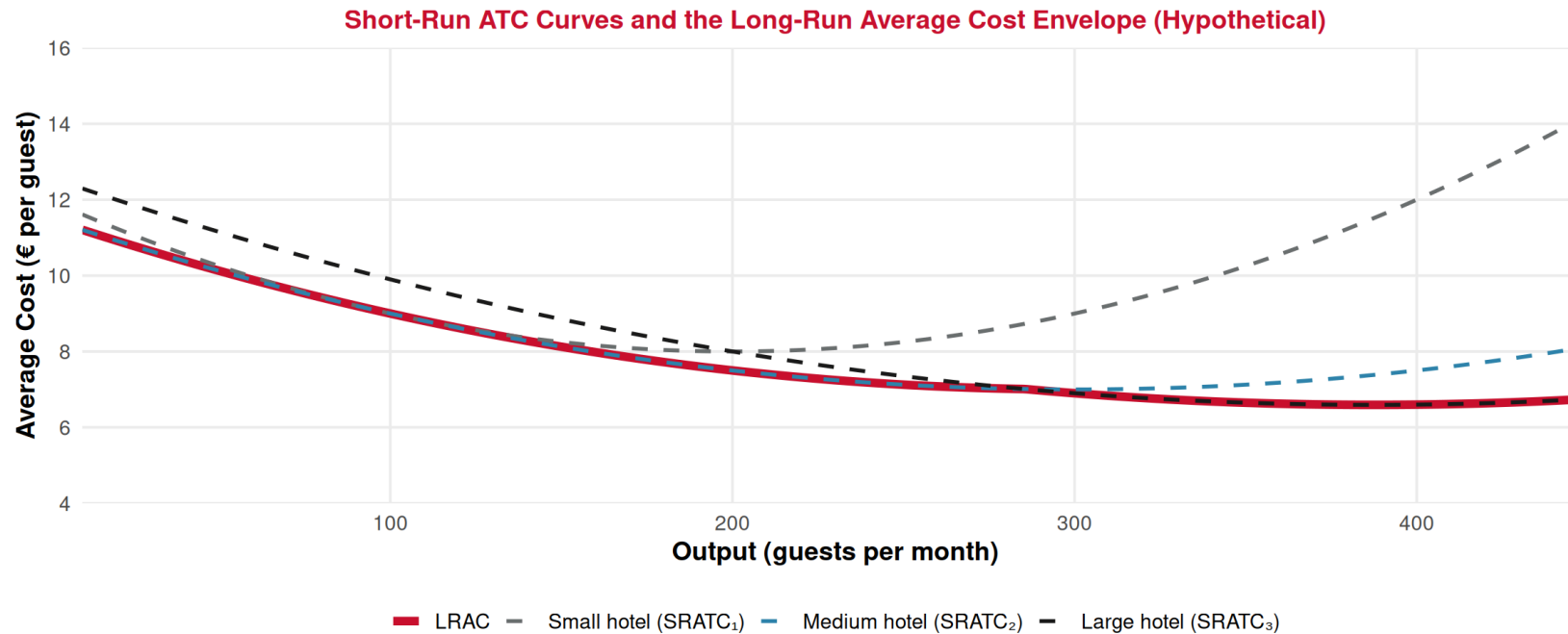
Think of it this way:

- **Short run**: “Given our current hotel (100 rooms), what’s our cost?”
- **Long run**: “What **size of hotel** minimizes our cost per guest?”

The LRAC curve is formed by choosing the **best short-run plant size** for each output level.

👉 The LRAC is the “**envelope**” of all possible short-run ATC curves.

The Envelope: LRAC as the Best of All Short Runs



Each dashed curve = one plant size (short run). The **bold red LRAC** = lowest cost achievable at each Q .

👉 In the long run, the firm **chooses the plant size** that minimizes cost for its desired output.

Economies and Diseconomies of Scale

The shape of the LRAC curve reveals three zones:

↓ Economies of Scale

LRAC is **falling**

Bigger → cheaper per unit

Tourism: Large hotel chains get bulk discounts, shared booking platforms, brand recognition

“Getting bigger makes us more efficient”

↔ Constant Returns

LRAC is **flat**

Doubling inputs doubles output (cost per unit stays the same)

Tourism: Mid-sized operators replicating a proven format across locations

“We’re at the right size”

↑ Diseconomies of Scale


LRAC is **rising**

Bigger → more expensive per unit

Tourism: Huge resorts with bureaucracy, coordination problems, slow decision-making

“We’ve grown too big”






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 **Minimum Efficient Scale (MES):** the smallest output where LRAC reaches its minimum. Firms should aim to operate **at or beyond** this point.






Economies of Scale in Tourism: Real-World Logic

Why do big hotel chains (Marriott, Accor, Hilton) have cost advantages?

Sources of economies of scale :

-  **Bulk purchasing**: negotiate lower prices with suppliers (linens, food, amenities)
-  **Technology**: one central booking system serves thousands of properties
-  **Marketing**: global advertising cost spread over many hotels
-  **Training**: standardized programs cheaper per employee
-  **Financing**: large firms borrow at lower interest rates

Why not grow forever?

-  **Bureaucracy**: decisions take longer in large organizations
 -  **Communication**: harder to coordinate across 8,000+ properties
 -  **Loss of local identity**: cookie-cutter hotels may lose appeal
 -  **Monitoring costs**: ensuring quality across locations
-  This is why **boutique hotels** can survive alongside chains — they operate efficiently at a smaller scale with a different value proposition.

Putting It All Together

Summary of Cost Curves

| Concept | Formula | Shape | Key Fact |
|--------------------------------|------------------------|-------------------------------|---|
| Fixed Cost (FC) | Constant | Horizontal line | Doesn't change with output |
| Variable Cost (VC) | Changes with Q | S-shaped (rises, accelerates) | Due to diminishing returns |
| Total Cost (TC) | $FC + VC$ | VC shifted up by FC | |
| Avg Fixed Cost (AFC) | FC / Q | Always declining | Spreading the overhead |
| Avg Variable Cost (AVC) | VC / Q | U-shaped | Min where MC crosses it |
| Avg Total Cost (ATC) | $TC / Q = AFC + AVC$ | U-shaped (above AVC) | Gap to AVC = AFC (shrinks) |
| Marginal Cost (MC) | $\Delta TC / \Delta Q$ | U-shaped | Crosses AVC & ATC at their mins |
| LRAC | Envelope of SRATCs | U-shaped | Economies \rightarrow constant \rightarrow diseconomies |

Summary

Today's Key Takeaways:

1. **Short run**: at least one input is fixed; **Long run**: all inputs are variable
2. **Total costs**: $TC = FC + VC$ — fixed costs don't depend on output
3. **Average costs**: $ATC = AFC + AVC$ — both per-unit measures
4. **Marginal cost**: the extra cost of one more unit — **the most important curve**
5. **MC crosses AVC and ATC at their minimums** (the “exam grade” rule)
6. **LRAC**: the envelope of short-run ATC curves — shows optimal plant size
7. **Economies of scale** explain why large hotel chains exist; **diseconomies** explain why they don't grow forever

Connection: Costs are the **foundation** of supply. Next lecture we use MC to find the **profit-maximizing output**.

Next (Lecture 11, March 20): Companies — Profit Maximization 💰

Exercises

Practice Time! 

Cost curves, short run, and long run.

Exercise 1: Multiple Choice

Question: A tour operator in Lisbon has monthly fixed costs of €10,000 (office lease, software) and variable costs that depend on the number of tours sold. Currently, they sell 200 tours per month with an ATC of €80 per tour. If they increase to 250 tours and ATC falls to €72 per tour, what can we conclude?

- A. Marginal cost of the additional tours is below €72
- B. Marginal cost of the additional tours is above €80
- C. The firm is experiencing diseconomies of scale
- D. Fixed costs have increased

Answer: A

If ATC is **falling** (from €80 to €72), it must be that MC is **pulling the average down**. From the “exam grade” analogy: if your average falls, the new grade must be **below** the current average. So $MC < ATC = €72$ for those additional 50 tours.

Exercise 2: Multiple Choice

Question: In the short run, a small hotel in Sintra has fixed costs of €6,000/month. When occupancy rises from 80 to 90 guests/month, total costs rise from €14,000 to €15,500. What is the marginal cost per additional guest?

- A. €150
- B. €155
- C. €172
- D. €175

Answer: A

$$MC = \frac{\Delta TC}{\Delta Q} = \frac{€15,500 - €14,000}{90 - 80} = \frac{€1,500}{10} = €150 \text{ per guest.}$$

Note: FC (€6,000) is **irrelevant** to MC — it doesn't change! Only variable costs drive the change in TC.

Exercise 3: Open Question

A small surf school in Ericeira has monthly fixed costs of €2,000 (equipment rental, insurance) and the following cost structure:

| Students/month (Q) | Variable Cost (VC) |
|--------------------|--------------------|
| 0 | €0 |
| 10 | €800 |
| 20 | €1,400 |
| 30 | €1,900 |
| 40 | €2,600 |
| 50 | €3,500 |
| 60 | €4,800 |

- Calculate TC, AFC, AVC, ATC, and MC for each output level.
- At what output level is AVC minimized? At what output is ATC minimized?
- Verify that $MC = AVC$ at the minimum of AVC, and $MC = ATC$ at the minimum of ATC.
- The school charges €90 per student. At what output level would you expect MC to equal this price? What does this tell us about the profit-maximizing output? (We'll formalize this in the next lecture!)
- In the long run, the school considers expanding to a second location. Explain, using the concept of economies of scale, under what conditions this expansion would lower the cost per student.

Exercise 3: Solution — Part a

| Q | FC | VC | TC | AFC | AVC | ATC | MC (per 10 students) |
|----|-------|-------|-------|--------|-------|--------|----------------------|
| 0 | 2,000 | 0 | 2,000 | — | — | — | — |
| 10 | 2,000 | 800 | 2,800 | 200.00 | 80.00 | 280.00 | 80.00 |
| 20 | 2,000 | 1,400 | 3,400 | 100.00 | 70.00 | 170.00 | 60.00 |
| 30 | 2,000 | 1,900 | 3,900 | 66.67 | 63.33 | 130.00 | 50.00 |
| 40 | 2,000 | 2,600 | 4,600 | 50.00 | 65.00 | 115.00 | 70.00 |
| 50 | 2,000 | 3,500 | 5,500 | 40.00 | 70.00 | 110.00 | 90.00 |
| 60 | 2,000 | 4,800 | 6,800 | 33.33 | 80.00 | 113.33 | 130.00 |

MC calculation example: From $Q = 20$ to $Q = 30$: $MC = \frac{3,900 - 3,400}{30 - 20} = \frac{500}{10} = €50$ per student.

Exercise 3: Solution — Parts b, c & d

b) AVC is minimized at $Q = 30$ (AVC = €63.33). ATC is minimized at $Q = 50$ (ATC = €110.00).





👉 AVC minimum comes **before** ATC minimum — this is always the case because AFC is still pulling ATC down even after AVC starts rising.

c) At $Q = 30$ (AVC minimum): MC just shifted from €60 (at $Q=20$) to €50 (at $Q=30$), and then rises to €70 (at $Q=40$). MC passes through the AVC minimum range, consistent with theory. At $Q = 50$ (ATC minimum): MC = €90, while ATC = €110. Between $Q=40$ and $Q=50$, MC (€90) is still below ATC (€110), pulling it down. At $Q=60$, MC = €130 > ATC = €113.33, so ATC starts rising. The crossing happens between $Q=50$ and $Q=60$. ✓

d) Price = €90. MC = €90 at $Q = 50$. This suggests the profit-maximizing output is around **50 students**. At this level, each additional student brings in exactly as much revenue (€90) as they cost to add. Producing more ($Q=60$) would mean MC = €130 > €90 = price → **losing money on each extra student**.


Exercise 3: Solution — Part e

e) Expanding to a second location would lower the cost per student if the school can achieve **economies of scale**:

-  **Shared equipment purchases**: buying boards and wetsuits in bulk at a discount
-  **Single booking/marketing system** serving both locations
-  **Instructor training**: standardized program developed once, used at both sites
-  **Brand recognition**: marketing costs spread over more students

However, expansion could also create **diseconomies** if:

- Coordination between locations is difficult (travel time, communication)
- Quality control drops when the owner can't supervise both sites
- Each beach has unique conditions requiring different approaches

 Expansion makes sense if the LRAC **falls** at the combined output — that is, if the firm is still in the **economies of scale** region.

Next Lecture

March 20, 2026: Companies — Profit Maximization 💰

How do firms use cost curves to choose the **optimal output**?

👉 **Spoiler:** The answer involves setting **Price = MC** in perfect competition!

Thank You!

Questions? 🙋

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Next class: Friday, March 20, 2026