Producer Theory

Lecture 14: Producer Surplus, Market Supply & Linear Supply

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Recap: Lecture 13 ⏪

What we covered last time:

- **The seller's supply rule**: Produce where $P = MC$ (rising portion, $P \geq AVC$)
- The individual supply curve = MC above AVC
- **Supply shifts**: input costs ↑ → supply shifts left; technology ↑ → supply shifts right
- Fixed costs don't shift the short-run supply curve

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🎯 **Today**: Three new pieces to complete the supply-side toolkit:

- 1️⃣ **Producer surplus** — the supply-side mirror of consumer surplus
- 2️⃣ **Market supply** — adding up individual firms' supply curves
- 3️⃣ **Linear supply** — the equation $P = c + dQ$ and how to work with it

Producer Surplus

The Intuition: What Do Producers Gain? 💰

In Lecture 8, we learned that **consumer surplus** measures the net benefit buyers get from a market — the gap between what they're **willing to pay** and what they **actually pay**.

PRODUCER SURPLUS (PS)

The net benefit producers get from selling in a market. It is the difference between the price they **receive** and the **minimum price** they would have been willing to accept (their marginal cost).

$$PS = \text{Total Revenue} - \text{Total Variable Cost} = TR - VC$$

Graphically: the area **above the supply curve** and **below the price line**.

💡 Producer surplus is **not** the same as profit! Profit = TR − TC = TR − VC − FC. Producer surplus = TR − VC. The difference is **fixed costs**:

$$PS = \pi + FC$$

PS vs Profit: Why the Difference Matters 🤔

Producer Surplus = $TR - VC$

- Measures the **gain from trading** in this market
- Does not subtract FC (which must be paid regardless)
- Equals the area **above the supply curve, below P**
- Always ≥ 0 (if the firm is operating)

Economic Profit = $TR - TC = TR - VC - FC$

- Measures the **bottom-line result** after all costs
- Subtracts both VC and FC
- Can be positive, zero, or negative
- $\pi = PS - FC$

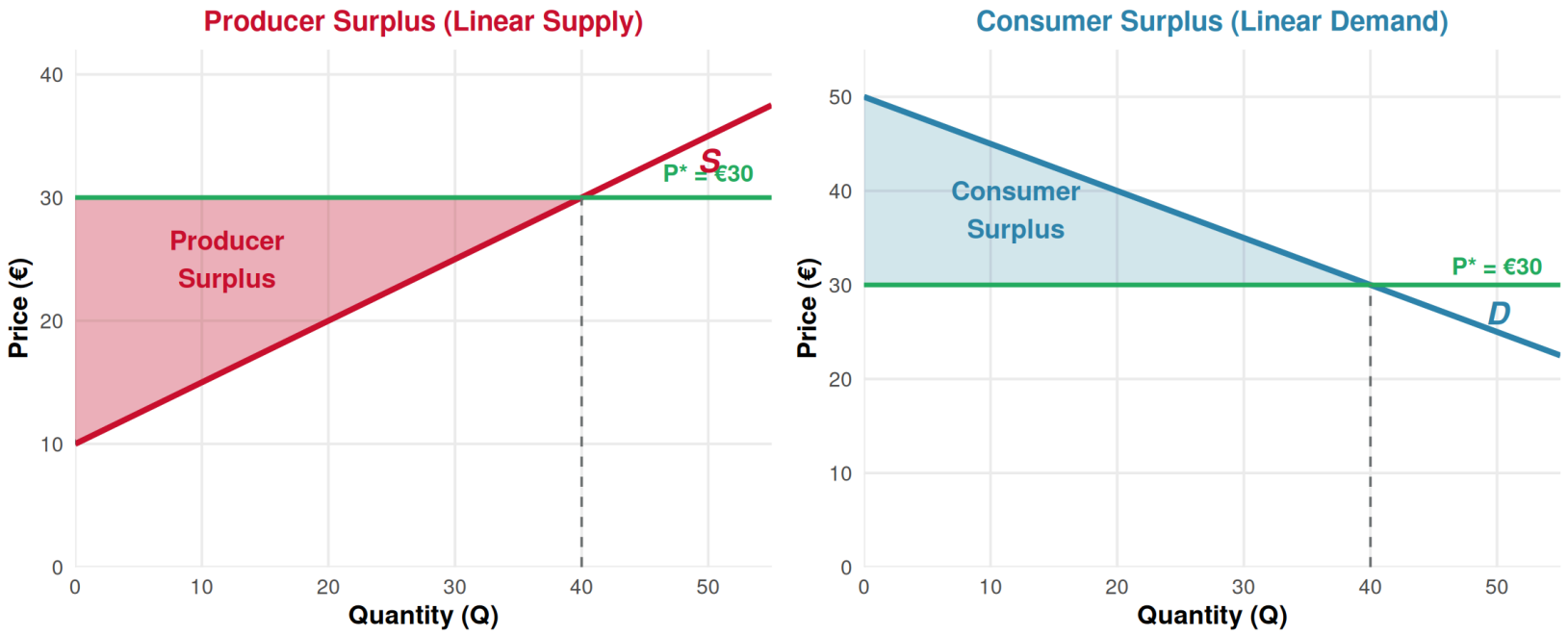
. . .

Example: A hotel earns TR = €50,000, has VC = €30,000 and FC = €25,000.

- $PS = 50{,}000 - 30{,}000 = €20{,}000$ ➡️ the hotel gains €20,000 from operating
- $\pi = 50{,}000 - 55{,}000 = -€5{,}000$ ➡️ the hotel makes a loss!
- But $PS > 0$ means the hotel is still **better off operating** (it covers VC and contributes €20,000 toward FC)

👉 This is exactly the **shutdown logic** from Lecture 12!

Producer Surplus Graphically



Left: PS = area above supply, below price = triangle. **Right**: CS = area below demand, above price = triangle (from Lecture 8). They are **mirror images**!

Calculating PS for Linear Supply ✏️

PRODUCER SURPLUS FORMULA (LINEAR SUPPLY)

$$PS = \frac{1}{2} \times Q^* \times (P^* - c)$$

where c is the **vertical intercept** of the supply curve (the minimum price at which any output is supplied) and P^* is the market price.

Example (from the graph): Supply is $P = 10 + 0.5Q$, market price $P^* = €30$.

At $P^* = 30$: $Q^* = \frac{30-10}{0.5} = 40$

$$PS = \frac{1}{2} \times 40 \times (30 - 10) = \frac{1}{2} \times 40 \times 20 = €400$$

👉 Compare with CS formula: $CS = \frac{1}{2} \times Q^* \times (b - P^*)$. Same triangle logic, just flipped!

What Does PS Tell Us in Tourism? ✈️

PS measures the net gain to producers from selling at the market price.

Tourism applications:

- 🏨 Higher hotel prices in peak season → **PS rises** (the area above supply expands)
- 🧾 A tourist tax → price received by hotel falls → **PS falls**
- 📈 New technology (lower MC) → supply shifts right → PS can rise even at the same price

Policy perspective:

- **Total welfare** = CS + PS
- Taxes/regulations reduce CS + PS (deadweight loss)
- PS helps measure the **impact on producers** of any policy change

👉 When Portugal debates a tourist tax, PS is one way to measure **how much hotels and tour operators lose**.

Market Supply

From Individual to Market Supply 🏭➡️🏛️

MARKET SUPPLY = HORIZONTAL SUM OF INDIVIDUAL SUPPLY CURVES

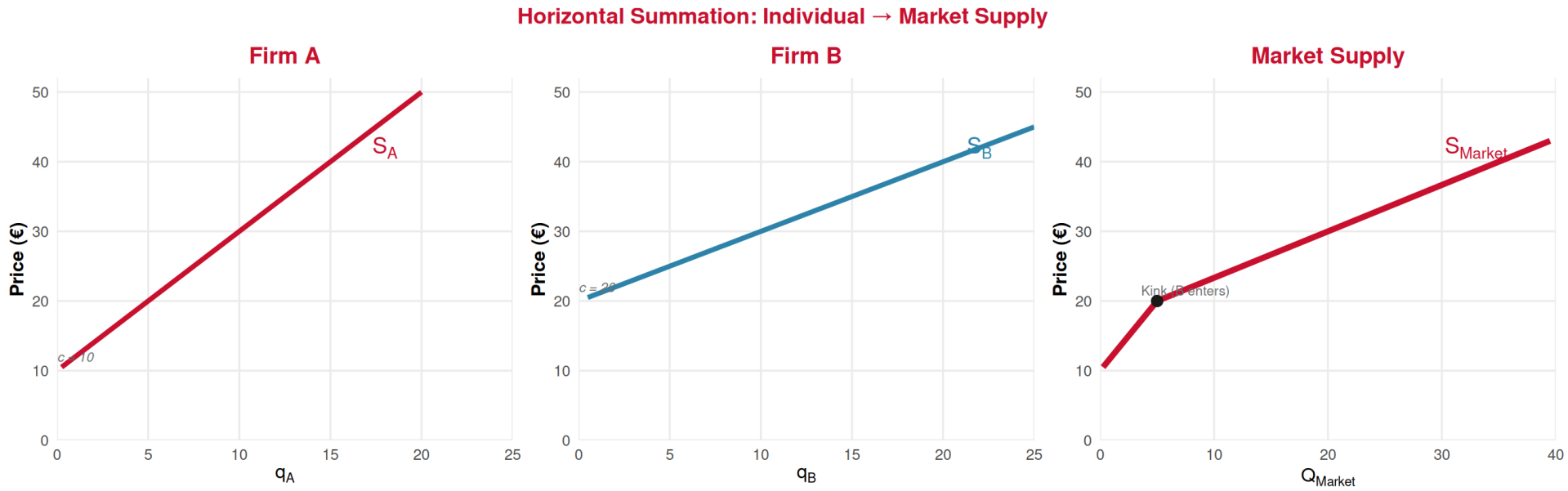
Just as we built market demand by adding individual demand curves **horizontally** (Lecture 8), we build market supply the same way: at each price, add up the quantities supplied by all firms.

The process:

1. At a given price P, each firm i supplies q_i (where $P = MC_i$)
2. **Market quantity** at that price: $Q_S = q_1 + q_2 + q_3 + \ldots$
3. Repeat for every price
4. Connect the dots → **market supply curve**

👉 If firms have different cost structures, the market supply curve may have **kinks** (just like market demand in Lecture 8!)

Horizontal Summation: Two Firms Example ➕



- For $P < 10$: no firm produces → $Q_S = 0$
- For $10 \leq P < 20$: only Firm A produces → $Q_S = q_A$
- For $P \geq 20$: both firms produce → $Q_S = q_A + q_B$ (kink at P = 20!)

👉 The **kink** occurs when Firm B's supply "turns on" — same logic as the demand kink in Lecture 8.

Building the Market Supply Equation 📏

From the previous example:

Firm A: $P = 10 + 2q_A \rightarrow q_A = \frac{P-10}{2}$ for $P \geq 10$

Firm B: $P = 20 + q_B \rightarrow q_B = P - 20$ for $P \geq 20$

Market supply (horizontal sum = add quantities at each price):

For $10 \leq P < 20$ (only A): $Q_S = q_A = \frac{P-10}{2}$

For $P \geq 20$ (A + B): $Q_S = \frac{P-10}{2} + (P - 20) = \frac{P-10+2P-40}{2} = \frac{3P-50}{2}$

...

WITH IDENTICAL FIRMS, IT'S SIMPLER!

If there are n identical firms each with supply $q = \frac{P-c}{d}$, then:

$$Q_S = n \times q = n \times \frac{P - c}{d}$$

The market supply has the **same intercept** but a **flatter slope** (more responsive to price).

Linear Supply

The Linear Supply Equation 📏

LINEAR SUPPLY — INVERSE FORM

$$P = c + dQ$$

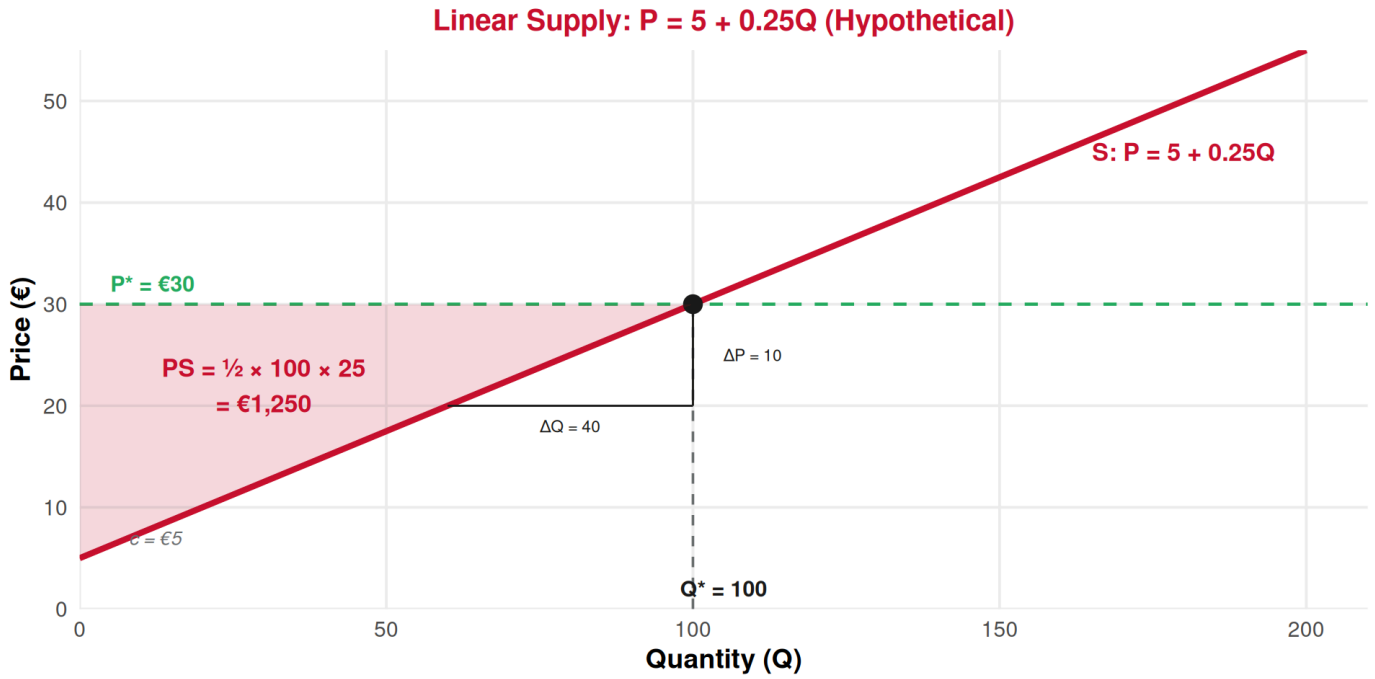
where P is price, Q is quantity supplied, c is the **vertical intercept**, and $d > 0$ is the **slope**.

Interpreting the parameters:

- c = the **minimum price** needed for any production (supply "turns on" at $P = c$)
- $d = \frac{\Delta P}{\Delta Q}$ (positive!) — how much price must rise to induce one more unit of output
- **Direct form**: $Q = \frac{P-c}{d}$ (useful for horizontal summation)

👉 Compare with demand: $P = b + mQ$ where $m < 0$. Supply has $d > 0$ — that's the **upward slope**!

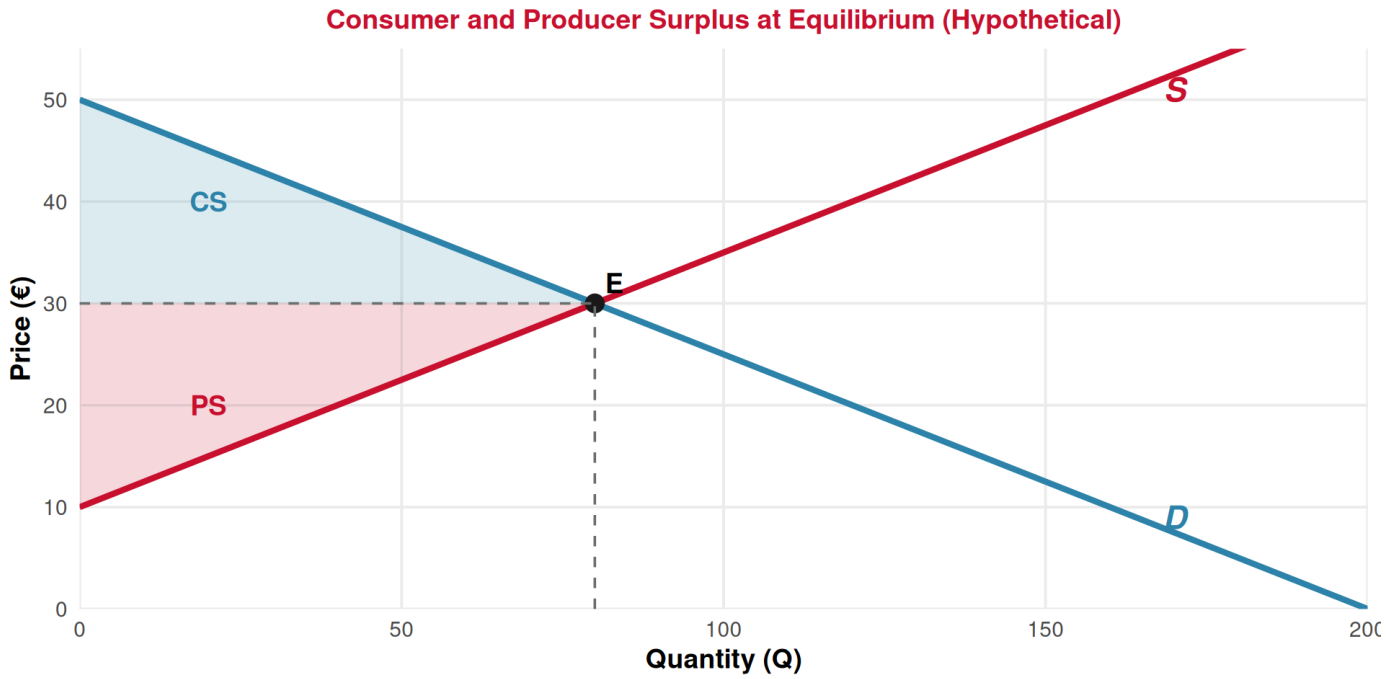
Reading a Linear Supply Curve 🔍



Supply: $P = 5 + 0.25Q$. At $P^* = €30$: $Q^* = \frac{30-5}{0.25} = 100$. Slope = $\frac{\Delta P}{\Delta Q} = \frac{10}{40} = 0.25$.

$PS = \frac{1}{2} \times 100 \times (30 - 5) = €1{,}250.$

Demand and Supply Together: The Big Picture 🤝



👉 **Total welfare** = CS + PS. At the competitive equilibrium, total welfare is **maximized** — there's no way to make one side better off without making the other worse off. We'll explore this more in Lecture 17 (market equilibrium).

Tourism Application

Tourism Market: CS and PS in Action 🌴

Imagine the market for guided tours in Lisbon:

Demand (tourists): $P = 50 - 0.25Q$

- Tourists are willing to pay up to €50 for a unique tour
- As price rises, fewer tourists book

Consumer surplus at equilibrium:

$$CS = \frac{1}{2} \times 80 \times (50 - 30) = €800$$

Tourists collectively gain €800 from buying tours below their willingness to pay.

Supply (tour operators): $P = 10 + 0.25Q$

- Tour guides need at least €10 to cover their minimum cost
- As more tours are offered, MC rises (hiring less experienced guides, longer hours)

Producer surplus at equilibrium:

$$PS = \frac{1}{2} \times 80 \times (30 - 10) = €800$$

Operators collectively gain €800 from selling tours above their minimum acceptable price.

. . .

👉 **Total welfare** = €800 + €800 = **€1,600**. This is the total gain from trade in this market!

Summary 📝

Today's Key Takeaways:

1. **Producer surplus** = $TR - VC$ = area above supply, below price. It measures the **gain to producers** from trading
2. PS ≠ profit! $\pi = PS - FC$. A firm can have positive PS but negative profit (and still rationally operate)
3. **PS for linear supply**: $PS = \frac{1}{2} \times Q^* \times (P^* - c)$ — a triangle, mirroring CS
4. **Market supply** = horizontal sum of individual supply curves (add quantities at each price)
5. With different firms, market supply may have **kinks** (when new firms "turn on" at higher prices)
6. **Linear supply**: $P = c + dQ$ — intercept c is the minimum supply price, slope $d > 0$
7. **Total welfare** = CS + PS, maximized at the competitive equilibrium
8. **Direct form** $Q = \frac{P-c}{d}$ is useful for horizontal summation

Connection: We now have both sides of the market: demand (Lectures 5–9) and supply (Lectures 10–14). Next lecture: **supply elasticity** — how sensitive is quantity supplied to price?

Next (Lecture 15, April 10): Calculation and Determinants of Supply Elasticity

Exercises

Practice Time! ✏️

Producer surplus, market supply, and linear supply.

Exercise 1: Multiple Choice

Question: A hotel has TR = €80,000/month, VC = €50,000/month, and FC = €35,000/month. What is its producer surplus and economic profit?

A. PS = €30,000, Profit = €30,000

B. PS = €30,000, Profit = −€5,000

C. PS = −€5,000, Profit = −€5,000

D. PS = €45,000, Profit = €10,000

Answer: B

$$PS = TR - VC = 80{,}000 - 50{,}000 = €30{,}000$$

$$\pi = TR - TC = 80{,}000 - (50{,}000 + 35{,}000) = 80{,}000 - 85{,}000 = -€5{,}000$$

The hotel has **positive PS** (worth operating!) but **negative profit** (making a loss). This is consistent: $PS = \pi + FC = -5{,}000 + 35{,}000 = €30{,}000$. ✅

Exercise 2: Multiple Choice

Question: The market supply for beach umbrella rentals in Cascais is $P = 2 + 0.1Q$ (€ per rental). At a market price of €12, what is the producer surplus?

A. €500

B. €600

C. €1,000

D. €1,200

Answer: A

At $P^* = 12$: $Q^* = \frac{12-2}{0.1} = 100$ rentals.

$$PS = \frac{1}{2} \times Q^* \times (P^* - c) = \frac{1}{2} \times 100 \times (12 - 2) = \frac{1}{2} \times 100 \times 10 = €500$$

Exercise 3: Open Question ✍️

The market for walking tours in Porto has two types of operators:

- **Professional guides** (20 identical firms): individual supply $P = 8 + 2q_P$ (€ per tour)
- **Freelance guides** (30 identical firms): individual supply $P = 15 + q_F$ (€ per tour)

a) Write the **individual** direct supply function (q as a function of P) for each type of guide.

b) At what price does each type "enter" the market (start supplying)?

c) Write the **market supply** function for each price range. (Hint: there will be a kink!)

d) At a market price of €25 per tour, how many tours does each type of guide offer? What is total market supply?

e) Calculate the **total producer surplus** at $P = €25$. (Calculate PS for each group separately, then sum.)

Exercise 3: Solution — Parts a & b

a) Direct supply functions (solve for q):

Professional: $P = 8 + 2q_P \rightarrow q_P = \frac{P-8}{2}$ for $P \geq 8$

Freelance: $P = 15 + q_F \rightarrow q_F = P - 15$ for $P \geq 15$

b) Each type "enters" at the vertical intercept of their supply:

- **Professionals** enter at $P = €8$ (lower costs, enter first)
- **Freelancers** enter at $P = €15$ (higher costs, enter later)

👉 The kink in market supply occurs at $P = €15$, where freelancers join the professionals.

Exercise 3: Solution — Parts c & d

c) Market supply (horizontal sum):

For $8 \leq P < 15$ (only professionals, 20 firms):

$$Q_S = 20 \times \frac{P - 8}{2} = 10(P - 8) = 10P - 80$$

For $P \geq 15$ (professionals + freelancers, 20 + 30 firms):

$$Q_S = 20 \times \frac{P - 8}{2} + 30 \times (P - 15) = 10P - 80 + 30P - 450 = 40P - 530$$

d) At $P = €25$:

- Each professional: $q_P = \frac{25-8}{2} = 8.5$ tours → 20 firms × 8.5 = **170 tours**
- Each freelancer: $q_F = 25 - 15 = 10$ tours → 30 firms × 10 = **300 tours**
- **Total market supply**: $Q_S = 170 + 300 = 470$ tours

✅ Check: $Q_S = 40(25) - 530 = 1000 - 530 = 470$ ✅

Exercise 3: Solution — Part e

e) Producer surplus at $P = €25$:

Professionals (20 firms): Each firm's PS = $\frac{1}{2} \times q_P \times (P - c_P) = \frac{1}{2} \times 8.5 \times (25 - 8) = \frac{1}{2} \times 8.5 \times 17 = €72.25$

Total PS for professionals: $20 \times 72.25 = €1{,}445$

Freelancers (30 firms): Each firm's PS = $\frac{1}{2} \times q_F \times (P - c_F) = \frac{1}{2} \times 10 \times (25 - 15) = \frac{1}{2} \times 10 \times 10 = €50$

Total PS for freelancers: $30 \times 50 = €1{,}500$

Total market PS = €1,445 + €1,500 = **€2,945**

💡 Professionals have a higher PS per firm (€72.25 vs €50) because their costs are lower — they capture more surplus from each sale. But freelancers contribute more in aggregate because there are more of them.

Next Lecture

April 10, 2026: Calculation and Determinants of Supply Elasticity 📈

How **sensitive** is quantity supplied to price changes?

👉 This mirrors Lecture 9 (demand elasticity) — the same formula, applied to supply!

Thank You!

Questions? 🙋

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Next class: Friday, April 10, 2026