

# Producer Theory

Lecture 15: Elasticity — Demand Recap & Supply Elasticity

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# Recap: Lecture 14

What we covered last time:

- **Producer surplus:**  $PS = TR - VC$  = area above supply, below price
- **Market supply:** horizontal sum of individual supply curves (kinks when firms enter)
- **Linear supply:**  $P = c + dQ$ , with  $PS = \frac{1}{2} \times Q^* \times (P^* - c)$

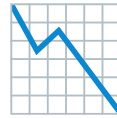
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 **Today:** How **responsive** are quantities to price changes?

Before we tackle **supply** elasticity, we need a quick recap of the **demand** elasticity concept — we ran out of time for it earlier in the course. The good news: the idea is the same on both sides!

# Demand Elasticity: The Key Ideas

# Price Elasticity of Demand



## PRICE ELASTICITY OF DEMAND ( $\epsilon_d$ )

The **percentage change in quantity demanded** resulting from a **1% change in price**:

$$\epsilon_d = \frac{\% \Delta Q_d}{\% \Delta P} = \frac{\Delta Q / Q}{\Delta P / P}$$

Since demand slopes downward ( $P \uparrow \rightarrow Q \downarrow$ ),  $\epsilon_d$  is **negative**. We often use the **absolute value**  $|\epsilon_d|$ .

| $ \epsilon_d $ | Classification             | Meaning                        | Tourism example                   |
|----------------|----------------------------|--------------------------------|-----------------------------------|
| $> 1$          | <b>Elastic</b>             | Q responds <b>more</b> than P  | Leisure beach holidays            |
| $= 1$          | <b>Unit elastic</b>        | Q responds <b>exactly</b> as P | — (theoretical midpoint)          |
| $< 1$          | <b>Inelastic</b>           | Q responds <b>less</b> than P  | Business travel                   |
| $= 0$          | <b>Perfectly inelastic</b> | Q doesn't respond at all       | Emergency medical flights         |
| $= \infty$     | <b>Perfectly elastic</b>   | Infinite response              | Identical commodity, many sellers |

# Calculating Demand Elasticity

Two methods (same as we'll use for supply):

**1** Point elasticity (at one point):

$$\varepsilon_d = \frac{\Delta Q}{\Delta P} \times \frac{P}{Q} = \frac{1}{\text{slope}} \times \frac{P}{Q}$$

**Example:** Demand is  $P = 50 - 2Q$ .

At  $P = 30$ ,  $Q = 10$ : slope =  $-2$

$$\varepsilon_d = \frac{1}{-2} \times \frac{30}{10} = -1.5$$

$|\varepsilon_d| = 1.5 > 1 \rightarrow$  **elastic**


**2** Arc elasticity (midpoint method):

$$\varepsilon_d = \frac{Q_2 - Q_1}{(Q_1 + Q_2)/2} \div \frac{P_2 - P_1}{(P_1 + P_2)/2}$$

**Example:** A museum raises price from €10 to €12. Visitors fall from 1,000 to 800.

$$\varepsilon_d = \frac{-200}{900} \div \frac{2}{11} = -1.22$$

$|\varepsilon_d| = 1.22 > 1 \rightarrow$  **elastic**

 Key insight for linear demand: elasticity **varies** along the curve — elastic at the top (high P, low Q), inelastic at the bottom (low P, high Q), unit elastic at the midpoint.

# The Revenue Test & Determinants

## The Revenue Test:

How does total revenue ( $TR = P \times Q$ ) respond to a price increase?

| Elasticity                             | P ↑ → TR?           | Intuition                 |
|--|---------------------|---------------------------|
| Elastic ( $ \varepsilon_d  > 1$ )      | TR <b>falls</b> ↓   | Q drops more than P rises |
| Unit elastic ( $ \varepsilon_d  = 1$ ) | TR <b>unchanged</b> | Effects cancel            |
| Inelastic ( $ \varepsilon_d  < 1$ )    | TR <b>rises</b> ↑   | Q drops less than P rises |

- 👉 Hotel prices up 10%, but bookings only drop 3%? Demand is inelastic → revenue **rises**!

## Five determinants of demand elasticity:

- 🔄 **Substitutes**: more substitutes → more elastic
- 💰 **Budget share**: bigger expense → more elastic
- 💎 **Necessity vs luxury**: luxuries more elastic
- 🕒 **Time horizon**: long run → more elastic
- 🌍 **Market definition**: narrower → more elastic

**Tourism**: Leisure travel is very elastic ( $|\varepsilon_d| \approx 2-3$ ), business travel is inelastic ( $|\varepsilon_d| \approx 0.3$ ). Airlines use this to price-discriminate!

# Demand Elasticity: Quick Practice

Quick check — try these mentally before revealing the answers!

**Q1:** Algarve resort demand:  $P = 500 - 0.05Q$ . At  $P = 100$ ,  $Q = 8,000$ . Is demand elastic or inelastic?

$$\varepsilon_d = \frac{1}{-0.05} \times \frac{100}{8000} = -20 \times 0.0125 = -0.25 \rightarrow \text{Inelastic } (|\varepsilon_d| = 0.25 < 1). \text{ Raising prices would increase revenue!}$$

**Q2:** If luxury cruise demand has  $|\varepsilon_d| = 2.5$  and operators raise prices by 8%, what happens to revenue?

Elastic demand  $\rightarrow$  price and revenue move in **opposite directions**. Revenue **falls**. ( $\% \Delta Q \approx -20\%$ , much larger than the 8% price rise.)

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 Now let's apply the **exact same logic** to the supply side!

# Price Elasticity of Supply

# Definition



## PRICE ELASTICITY OF SUPPLY ( $\epsilon_s$ )

The **percentage change in quantity supplied** resulting from a **1% change in price**:

$$\epsilon_s = \frac{\% \Delta Q_s}{\% \Delta P} = \frac{\Delta Q / Q}{\Delta P / P}$$

### Key differences from demand elasticity:

bulb: The formula is **identical** — the only difference is that supply slopes upward, so  $\epsilon_s > 0$ .

|                               | Demand elasticity ( $\epsilon_d$ )                 | Supply elasticity ( $\epsilon_s$ )                      |
|-------------------------------|--|---|
| <b>Sign</b>                   | Negative ( $P \uparrow \rightarrow Q \downarrow$ ) | <b>Positive</b> ( $P \uparrow \rightarrow Q \uparrow$ ) |
| <b>We use absolute value?</b> | Yes ( $ \epsilon_d $ )                             | No — it's already positive                              |
| <b>Interpretation</b>         | Responsiveness of buyers                           | Responsiveness of sellers                               |

# Classification



| Elasticity value         | Classification             | Meaning   |
|--------------------------|----------------------------|---|
| $\varepsilon_s > 1$      | <b>Elastic</b>             | Quantity supplied responds <b>more</b> than proportionally to price |
| $\varepsilon_s = 1$      | <b>Unit elastic</b>        | Quantity supplied responds <b>exactly</b> proportionally            |
| $\varepsilon_s < 1$      | <b>Inelastic</b>           | Quantity supplied responds <b>less</b> than proportionally          |
| $\varepsilon_s = 0$      | <b>Perfectly inelastic</b> | Quantity doesn't respond at all (vertical supply)                   |
| $\varepsilon_s = \infty$ | <b>Perfectly elastic</b>   | Any quantity at one fixed price (horizontal supply)                 |

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## Tourism intuition:

- 🏠 Hotel rooms in Lisbon in the short run → **inelastic** (can't build new rooms overnight)
- 🍴 Street food vendors in a tourist area → **elastic** (easy to set up a new stall)
- 🏛️ UNESCO heritage sites → **perfectly inelastic** (supply is fixed!)

# Calculating Supply Elasticity

# Point and Arc Methods

## 1 Point elasticity:

$$\varepsilon_s = \frac{1}{\text{slope}} \times \frac{P}{Q}$$

**Example:** Supply is  $P = 5 + 0.25Q$ . At  $P = 30$ ,  $Q = 100$ :

Slope = 0.25, so  $\frac{1}{\text{slope}} = 4$

$$\varepsilon_s = 4 \times \frac{30}{100} = 1.2$$

**Elastic** — a 1% price increase leads to a 1.2% increase in quantity supplied.

👉 Same methods as demand elasticity — just applied to the supply curve!

## 2 Arc elasticity (midpoint):

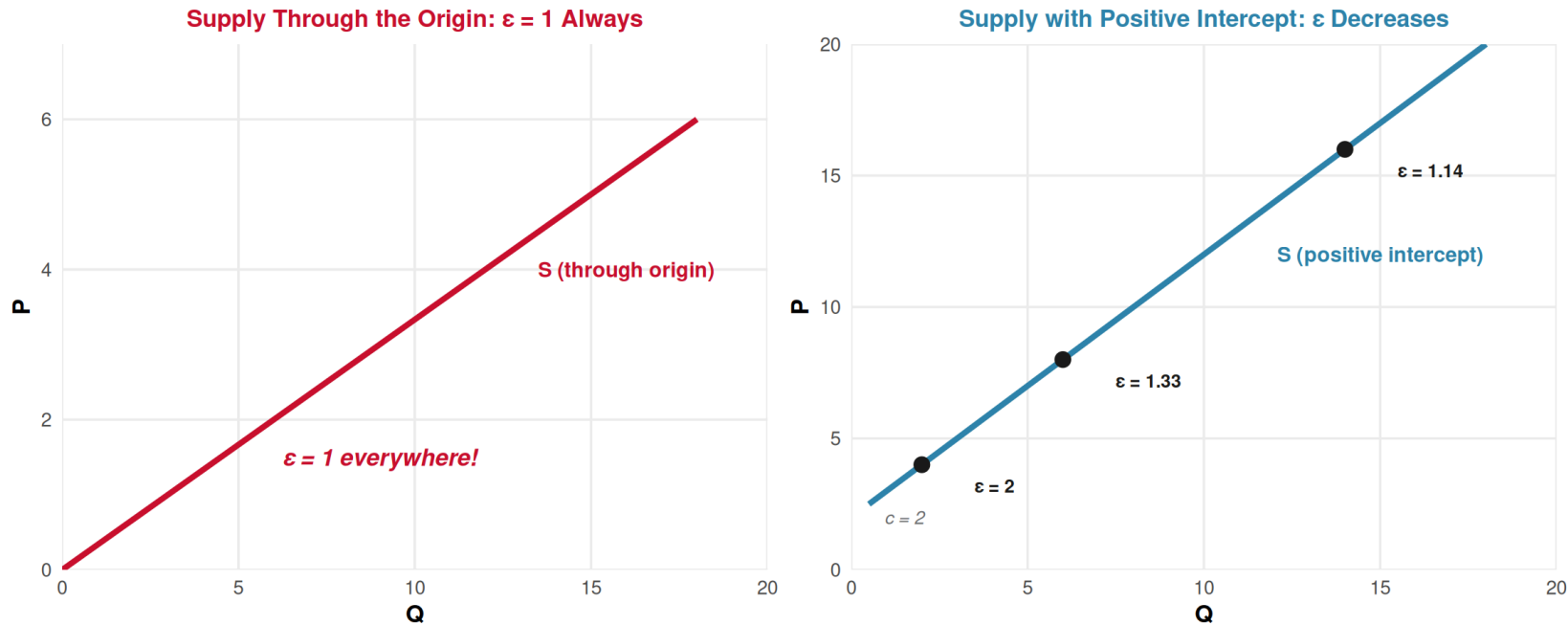
$$\varepsilon_s = \frac{Q_2 - Q_1}{(Q_1 + Q_2)/2} \div \frac{P_2 - P_1}{(P_1 + P_2)/2}$$

**Example:** Tour operator increases tours from 40 to 50/month when price rises from €80 to €100.

$$\varepsilon_s = \frac{10}{45} \div \frac{20}{90} = 0.222 \div 0.222 = 1.0$$

**Unit elastic** — quantity and price change by the same percentage.

# Elasticity Along a Linear Supply Curve



**Left:** Supply through the origin  $\rightarrow \epsilon_s = 1$  at **every** point (P and Q always change proportionally).

**Right:** Supply with positive intercept ( $c > 0$ )  $\rightarrow \epsilon_s > 1$  near the intercept, **falling** as Q increases.

# The Origin Rule & Quick Guide

Why does supply through the origin always have  $\varepsilon_s = 1$ ?

$$\text{If } P = dQ: \quad \varepsilon_s = \frac{1}{d} \times \frac{P}{Q} = \frac{1}{d} \times \frac{dQ}{Q} = 1$$

The  $P/Q$  ratio always equals  $d$ , which cancels perfectly with  $1/d$ .

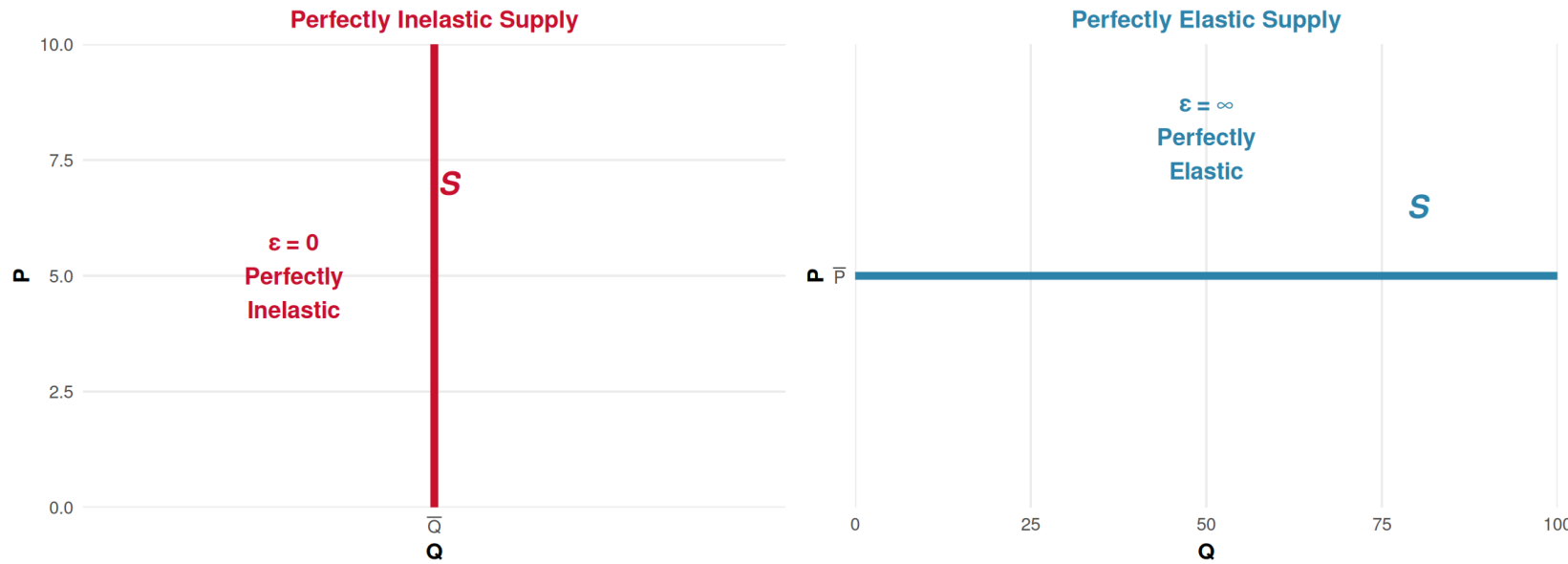
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## QUICK ELASTICITY GUIDE FOR LINEAR SUPPLY $P = c + dQ$

| Intercept                    | Elasticity                             | Shortcut formula                                 |
|------------------------------|--|--|
| $c = 0$ (through origin)     | $\varepsilon_s = 1$ everywhere         | —  |
| $c > 0$ (positive intercept) | $\varepsilon_s > 1$ , falling toward 1 | $\varepsilon_s = \frac{P}{P-c}$                  |
| $c < 0$ (hits Q-axis)        | $\varepsilon_s < 1$ everywhere         | $\varepsilon_s = \frac{P}{P-c}$ (where $c < 0$ ) |

 **Contrast with demand:** demand elasticity varies from  $\infty$  to 0 along a linear curve. Supply elasticity is more “stable.”

# Extreme Cases



**Perfectly inelastic** ( $\epsilon_s = 0$ ): Quantity fixed regardless of price.

*Tourism:* Hotel rooms in a fully built-up area, UNESCO sites, beach space.

**Perfectly elastic** ( $\epsilon_s = \infty$ ): Any quantity at one price — constant MC.

*Tourism:* Digital products (e-tickets, online guides) — near-zero MC per copy.



# Determinants of Supply Elasticity

# What Makes Supply More or Less Elastic? 🤔

The textbook identifies **four key determinants** (plus time):



## 1 Flexibility of production factors

Can inputs be used for other things? If yes → elastic.

-  **Elastic**: Unskilled labor (waiters can switch from retail to tourism)
-  **Inelastic**: Specialized skills (brain surgeons, master sommeliers)



## 2 Mobility of production factors

Can inputs move between locations? If yes → elastic.

-  **Elastic**: Touring musicians, portable equipment
-  **Inelastic**: Beachfront land, historical buildings

## 3 Ability to produce substitute inputs

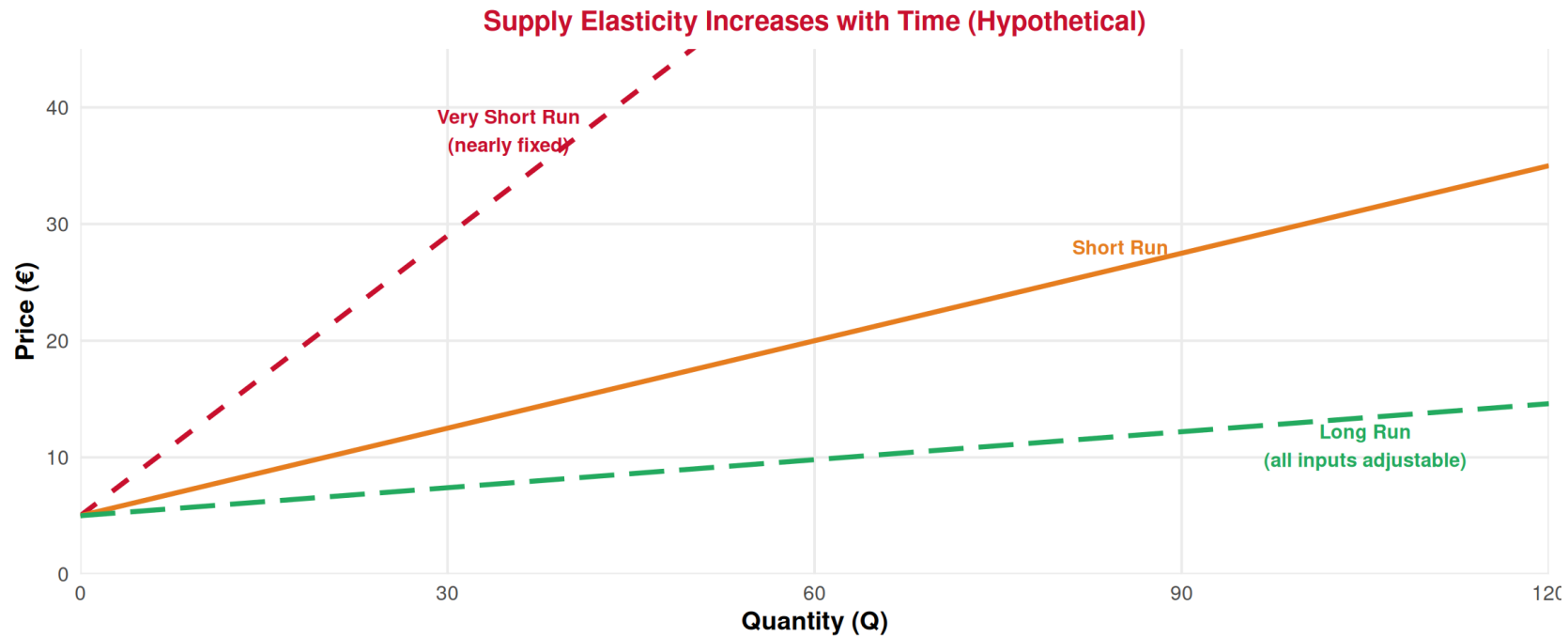
Can inputs be replicated over time? If yes → elastic.

-  **Elastic**: Train more tour guides, build more hotel rooms
-  **Inelastic**: Rare natural resources (diamond mines, volcanic hot springs)

## 4 Time horizon ⌚

- **Short run**: At least one input is fixed → more **inelastic**
- **Long run**: All inputs adjustable → more **elastic**
- **Very short run** (market period): supply is nearly **vertical**

# Time and Supply Elasticity






The **longer the time horizon**, the more firms can adjust all inputs → supply becomes **flatter** (more elastic).

# Tourism Applications

# Elasticity in Tourism: Both Sides

## Inelastic supply ( $\varepsilon_s$ small):




-  **Hotel rooms in central Lisbon:** limited space, takes years to build
-  **Museum capacity:** Jerónimos can't add rooms
-  **Beach access in Algarve:** fixed coastline

**Implication:** When demand rises (peak season), price rises **sharply** because supply can't respond much.

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-  **Both elasticities together** determine **who bears the burden** of taxes and price shocks! The side with **less elastic** response absorbs more of the burden.

## Elastic supply ( $\varepsilon_s$ large):

-  **Tour buses:** can add more buses relatively quickly
-  **Food trucks at festivals:** quick to set up
-  **Online booking services:** near-zero cost to scale

**Implication:** When demand rises, quantity adjusts more than price — price stays relatively stable.

# The Full Parallel: Demand vs Supply Elasticity

| Concept             | Demand Elasticity   | Supply Elasticity                                       |
|---------------------|---|---|
| Formula             | $\varepsilon_d = \frac{\Delta Q/Q}{\Delta P/P}$               | $\varepsilon_s = \frac{\Delta Q/Q}{\Delta P/P}$         |
| Sign                | Negative (use $ \varepsilon_d $ )                             | <b>Positive</b>   |
| Elastic             | $ \varepsilon_d  > 1$   | $\varepsilon_s > 1$                                     |
| Revenue test        | Elastic: $P \uparrow \rightarrow TR \downarrow$               | No direct revenue test                                  |
| Along linear curve  | Varies: $\infty \rightarrow 1 \rightarrow 0$                  | More stable; through origin = 1                         |
| Key determinants    | Substitutes, budget share, necessity, time, market definition | <b>Input flexibility, mobility, replicability, time</b> |
| Perfectly inelastic | Vertical demand   | Vertical supply   |
| Perfectly elastic   | Horizontal demand   | Horizontal supply                                       |

- 👉 Both sides use the **same formula** and **same classification**. The determinants differ because they reflect different underlying decisions (buying vs producing).

# Summary


## Today's Key Takeaways:

### Demand elasticity recap:

1.  $\varepsilon_d = \frac{\% \Delta Q}{\% \Delta P}$  — negative, use  $|\varepsilon_d|$ . Elastic  $> 1$ , inelastic  $< 1$
2. **Revenue test:** elastic  $\rightarrow P \uparrow$  means  $TR \downarrow$ ; inelastic  $\rightarrow P \uparrow$  means  $TR \uparrow$
3. **Determinants:** substitutes, budget share, necessity/luxury, time, market definition

### Supply elasticity (new):

4.  $\varepsilon_s = \frac{\% \Delta Q_s}{\% \Delta P}$  — always positive. Same formula, same classification
5. **Linear supply through origin:**  $\varepsilon_s = 1$  everywhere. Positive intercept:  $\varepsilon_s > 1$
6. **Determinants:** flexibility, mobility, and replicability of inputs, **time horizon**
7. **Time:** very short run (inelastic)  $\rightarrow$  short run  $\rightarrow$  long run (elastic)
8. Both elasticities together determine **price volatility** and **tax burden sharing**

**Next (Lecture 16, April 16):** Short Run and Long Run Market Equilibrium 

# Exercises

Practice Time! 

Demand and supply elasticity calculation.

# Exercise 1: Multiple Choice

**Question:** The supply of hotel rooms in Porto's city center has  $\varepsilon_s = 0.3$ . If room prices increase by 20%, what happens to the quantity of rooms supplied?

- A. Quantity increases by 60%
- B. Quantity increases by 6%
- C. Quantity increases by 0.6%
- D. Quantity increases by 20%

**Answer: B**

$$\% \Delta Q_s = \varepsilon_s \times \% \Delta P = 0.3 \times 20\% = 6\%$$

Supply is **inelastic** ( $\varepsilon_s = 0.3 < 1$ ): even a large price increase (20%) produces only a small quantity response (6%). You can't build new hotel rooms quickly!

## Exercise 2: Multiple Choice

**Question:** Demand for TAP flights from Lisbon to London has  $|\varepsilon_d| = 1.8$ . If TAP raises fares by 10%, what happens to total revenue from this route?

- A. Revenue increases by 18%
- B. Revenue increases because higher price means more money per ticket
- C. Revenue falls because passenger numbers drop by more than 10%
- D. Revenue stays the same

**Answer: C**

Demand is **elastic** ( $|\varepsilon_d| = 1.8 > 1$ ): a 10% fare increase causes approximately  $1.8 \times 10\% = 18\%$  fewer passengers. Since quantity drops **proportionally more** than price rises,  $TR = P \times Q$  **falls**. This is the **revenue test** for elastic demand.

# Exercise 3: Open Question

The market for *tuk-tuk* tours in Lisbon has the following estimated curves:

- **Demand:**  $P = 36 - 0.2Q$  (€ per tour)
  - **Supply:**  $P = 6 + 0.1Q$  (€ per tour)
- a) Find the equilibrium price and quantity.
  - b) Calculate the **price elasticity of demand** at the equilibrium point. Is demand elastic or inelastic?
  - c) Calculate the **price elasticity of supply** at the equilibrium point. Is supply elastic or inelastic?
  - d) At the equilibrium, if the city council imposes a small price increase (say €1), would you expect total revenue to rise or fall? Use the revenue test from part (b).
  - e) The supply has a positive intercept ( $c = 6$ ). Using the shortcut formula  $\varepsilon_s = \frac{P}{P-c}$ , verify your answer from part (c). At what price would supply elasticity equal exactly 2?
  - f) The city council considers a €3 per-tour tax on operators. This shifts supply to  $P = 9 + 0.1Q$ . Find the new equilibrium. Who bears more of the tax burden — tourists or operators? Relate your answer to the elasticities you calculated.

## Exercise 3: Solution — Parts a, b & c

a) Equilibrium:  $36 - 0.2Q = 6 + 0.1Q \rightarrow 30 = 0.3Q \rightarrow Q^* = 100, P^* = 36 - 0.2(100) = \text{€}16$

b) Demand elasticity at  $(Q^* = 100, P^* = 16)$ :

Demand slope =  $-0.2$ , so  $\frac{1}{\text{slope}} = \frac{1}{-0.2} = -5$

$$\varepsilon_d = -5 \times \frac{16}{100} = -0.8$$

$|\varepsilon_d| = 0.8 < 1 \rightarrow$  Demand is **inelastic** at the equilibrium.

c) Supply elasticity at  $(Q^* = 100, P^* = 16)$ :


Supply slope =  $0.1$ , so  $\frac{1}{\text{slope}} = 10$

$$\varepsilon_s = 10 \times \frac{16}{100} = 1.6$$

$\varepsilon_s = 1.6 > 1 \rightarrow$  Supply is **elastic** at the equilibrium.

## Exercise 3: Solution — Parts d & e

**d)** Since demand is **inelastic** ( $|\varepsilon_d| = 0.8 < 1$ ), the revenue test says: a price increase  $\rightarrow$  total revenue **rises**. Quantity falls, but by a smaller percentage than price rises. The €1 increase would increase TR.

**e)** Shortcut:  $\varepsilon_s = \frac{P}{P-c} = \frac{16}{16-6} = \frac{16}{10} = 1.6$   Matches part (c)!

For  $\varepsilon_s = 2$ :  $\frac{P}{P-6} = 2 \rightarrow P = 2P - 12 \rightarrow P = €12$

At  $P = 12$ :  $Q = \frac{12-6}{0.1} = 60$  tours. So supply is more elastic at **lower prices** (closer to the intercept).

# Exercise 3: Solution — Part f

f) New supply with €3 tax:  $P = 9 + 0.1Q$

New equilibrium:  $36 - 0.2Q = 9 + 0.1Q \rightarrow 27 = 0.3Q \rightarrow Q^* = 90, P^* = 36 - 0.2(90) = €18$

|  | Before tax | After tax      | Change |
|--|------------|----------------|--------|
| Price paid by tourists                   | €16        | €18            | +€2    |
| Price received by operators (net of tax) | €16        | €18 - €3 = €15 | -€1    |
| Quantity                                 | 100        | 90             | -10    |

Of the €3 tax: tourists pay **€2 more**, operators receive **€1 less**.

👉 **Tourists bear more** (€2 out of €3) because **demand is more inelastic** ( $|\varepsilon_d| = 0.8$ ) than supply is ( $\varepsilon_s = 1.6$ ). The side with **less elastic** response absorbs more of the tax.

💡 This is a **general rule**: tax burden falls more heavily on the side of the market that is **less responsive** to price changes.

## Next Lecture

**April 16, 2026:** Short Run and Long Run Market Equilibrium 

We finally put **demand and supply together** to find prices and quantities!

👉 This is the payoff of Lectures 5–15: consumer theory meets producer theory!

# Thank You!

Questions? 🙋

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*Next class: Thursday, April 16, 2026*